

Precision Measurements of the weak mixing angle

$$\sin^2 \theta_{eff}^l (M_Z) \text{ and } \sin^2 \theta_W^{\overline{MS}} (\mu)$$

A new era of Precision Electroweak Measurements at the LHC

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MSU High Energy Seminar

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Zoom: <https://msu.zoom.us/j/8295551623>

Abstract

In the Standard Model (SM), the effective leptonic weak mixing angle $\sin^2 \theta_{\text{eff}}^\ell$ and the mass of the W boson can be predicted to very high accuracy $\sin^2 \theta_{\text{eff}}^\ell = 0.23152 \pm 0.00006$, and $M_W = 80.3570 \pm 0.0060$

by using the experimental values of the mass of the Z boson (M_Z), the Fermi constant, the fine structure constant, the mass of Higgs boson, the quark masses, and the value of the Quantum Chromodynamics (QCD) strong coupling (α_s) at M_Z .

Therefore, deviations of measurements from SM predictions can be indications of new physics.

With recent advances in experimental and analysis techniques, measurements of $\sin^2 \theta_{\text{eff}}^\ell$ at the Large Hadron Collider (LHC), extracted from the forward-backward asymmetry (A_{FB}) in Drell-Yan dilepton events are now competitive with previously published measurements at e^+e^- collider experiments.

We report on a more precise extraction of the effective leptonic weak mixing angle via an updated analysis of published CMS Drell-Yan A_{FB} measurements at 13 TeV. With additional PDF constraints from measurements of the W decay lepton charge asymmetry at 13 TeV and the W/Z cross section ratios we obtain

$$\sin^2 \theta_{\text{eff}}^\ell = 0.23154 \pm 0.00024, \quad (\text{In which the PDF contribution is } 0.00016)$$

which is in excellent agreement with the SM 2026 indirect prediction of 0.23152 ± 0.00006 .

A new era of Precision Electroweak Measurements at the LHC: How was it done, and what is the outlook for the future

Weak mixing angle

- *Electroweak on-shell mixing angle* - $\sin^2 \theta_W$
- A fundamental parameter in the Standard model
 - Related to the unification of the electromagnetic and weak forces

$$\sin^2 \theta_W (\text{on shell}) = 1 - \left(\frac{m_W}{m_Z} \right)^2$$

- Effective **leptonic** mixing angle – $\sin^2 \theta_{\text{eff}}^l (M_Z)$

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(M_Z^2, \sin^2 \theta_W)] \sin^2 \theta_W$$

$$\hookrightarrow \approx 1.0363$$

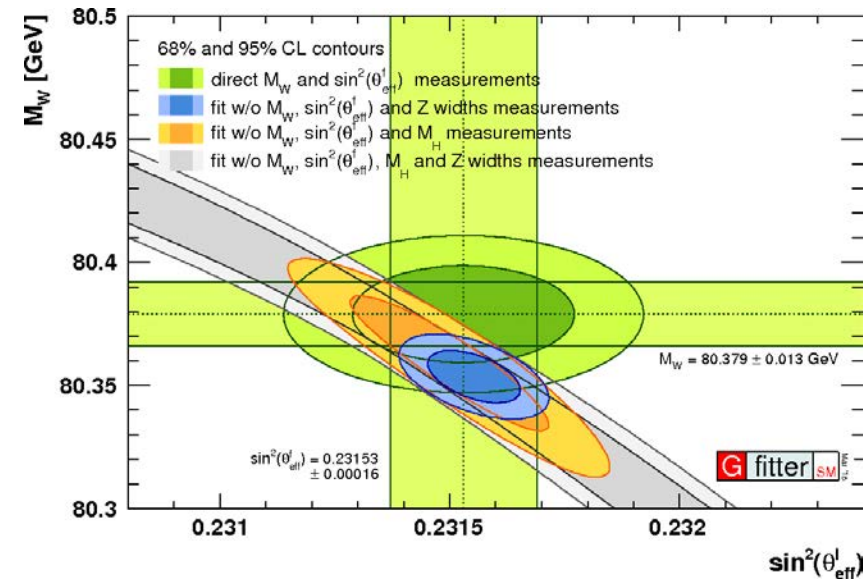
- **Includes loop corrections**
- Reflects the effective couplings of the Z boson to leptons

- Measurement of $\sin^2 \theta_{\text{eff}}^l (M_Z)$

- Provides a consistency check of the Standard Model
- **Serves as a powerful tool for searching for new phenomena**
(e.g. through loop corrections from high mass particles)

± 0.00024 in $\sin^2 \theta_W^{\text{eff}}$ is equivalent to indirect determination of M_W to ± 12 MeV

Gfitter (Aug 2024)



Additional definitions of the weak mixing angle

- Effective leptonic mixing angle

$$\sin^2 \theta_{\text{eff}}^l = \frac{1}{4} \left[1 - \text{Re} \left(\frac{g_V^l}{g_A^l} \right) \right]$$

- g_V^l and g_A^l : effective vector and axial-vector couplings of the Z boson to charged leptons
- At the Z pole, experiments are most directly sensitive to the effective Zll coupling ratio
- $\sin^2 \theta_{\text{eff}}^l$ is the natural pseudo-observable

$\sin^2 \theta_{\text{eff}}^l$ **has small dependence on scale (dilepton mass for DY) around the Z pole region**

- $\overline{\text{MS}}$ mixing angle

$$\sin^2 \theta_W^{\overline{\text{MS}}}(\mu) = \frac{\hat{g}'^2(\mu)}{\hat{g}^2(\mu) + \hat{g}'^2(\mu)}$$

- $\hat{g}^2(\mu)$ and $\hat{g}'^2(\mu)$ are the running SU(2) and U(1) gauge couplings renormalized in the $\overline{\text{MS}}$ scheme
- Away from the Z pole, the extracted angle is more process dependent due to radiative corrections
- $\overline{\text{MS}}$ angle provides a common reference for comparing measurements performed at different energy scales

$\sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$ **has significant dependence on scale (dilepton mass for DY)**

- $\sin^2 \theta_{\text{eff}}^l - \sin^2 \theta_W^{\overline{\text{MS}}}(m_Z) \cong 0.00032$
- For high-mass DY measurements, we will use the $\overline{\text{MS}}$ weak mixing angle evaluated at $\mu = m_{ll}$

References: Scale dependence of $-\sin^2 \theta_{\text{eff}}^l(\mu)$ vs $\sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$

$$\sin^2 \theta_{\text{eff}}^l(\mu)$$

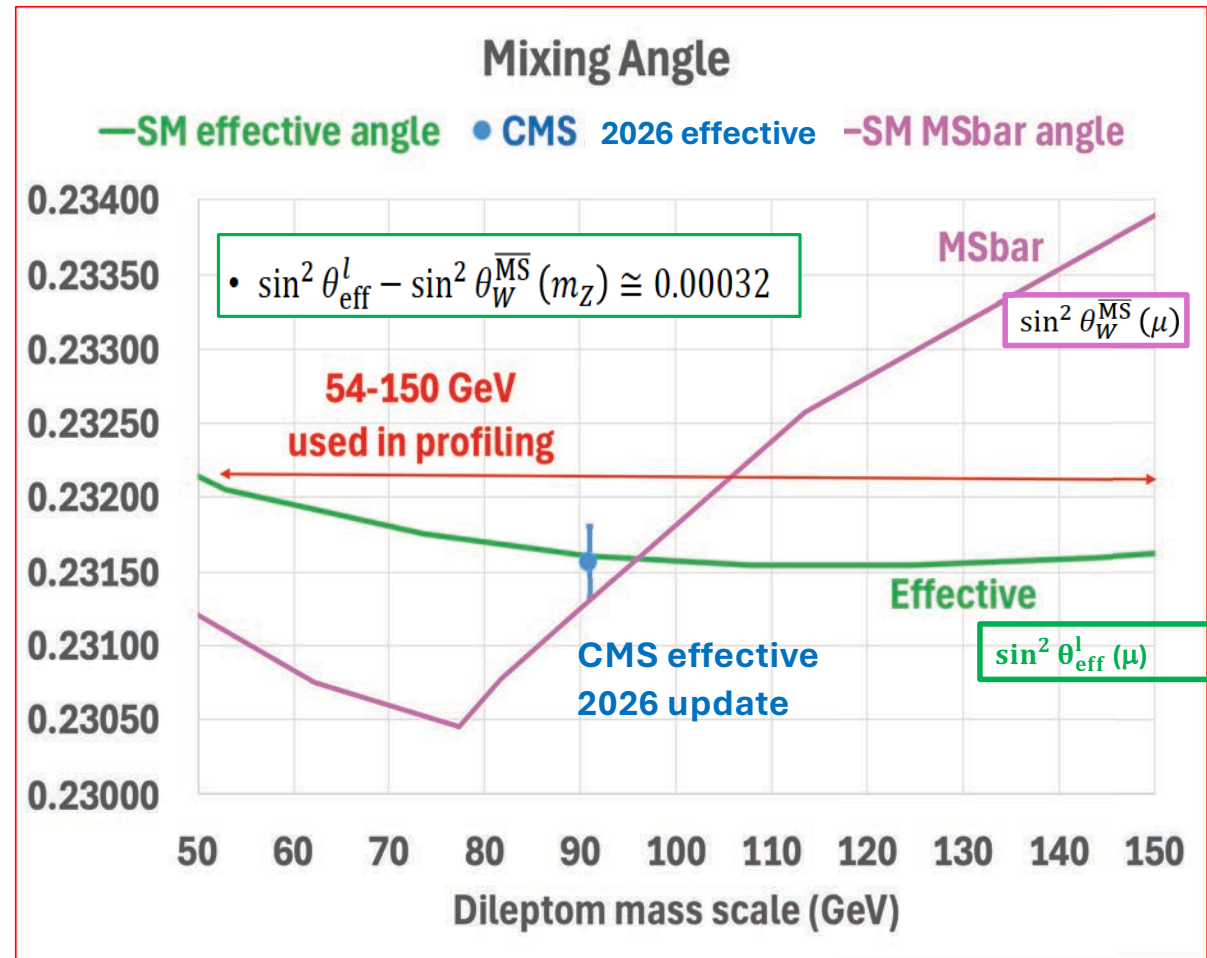
Effective angle has small variation with scale see:
Moller scattering asymmetries. Andrzej Czarnecki,
 William J. Marciano Int.J.Mod.Phys.A 15 (2000) 2365-
 2376. e-Print: [hep-ph/0003049](https://arxiv.org/abs/hep-ph/0003049) [hep-ph]

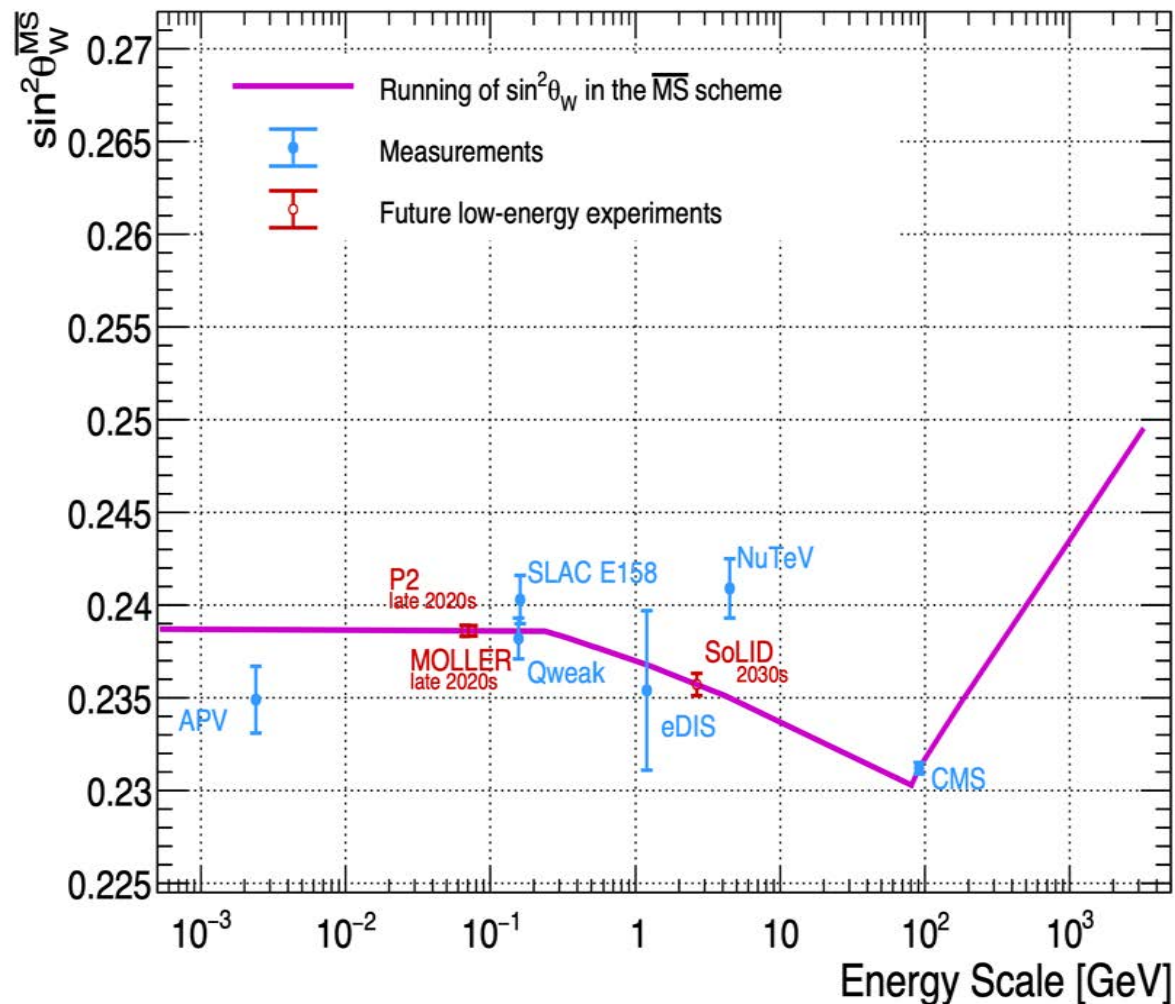
$$\sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$$

MSbar angle has Large variation with scale see:
*Implications of Recent Experimental and Theoretical
 Results on Electroweak Precision Tests.* Jens Erler.
 PoS CORFU2024 (2025) 235 e-Print: [2505.03457](https://arxiv.org/abs/2505.03457)
 [hep-ph]

$$\sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$$

MC study of measurements above Z pole
See: *Probing the weak mixing angle at high energies at
 the LHC and HL-LHC.* Simone Amoroso, Mauro
 Chiesa, Clara Lavinia Del Pio, Katerina Lipka, Fulvio
 Piccinini, Phys.Lett.B 844 (2023) 138103 e-Print:
[2302.10782](https://arxiv.org/abs/2302.10782) [hep-ph]





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Measurements at different energy scales

$$\sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$$

- Low-energy measurements
 - Several measurements with limited precision (~ 0.001)
 - Some future experiments with high precision (~ 0.0003)
- Z-pole measurements
 - LEP, SLD, Tevatron, LHC
 - CMS SMP-22-010
 - ~ 0.0003
- High-energy measurements
 - No measurement yet

High-energy measurement in Run2+Run3 Currently under analysis at CMS by Rochester-SNU group discussed later in this talk.

APV, Qweak, E158, eDIS, NuTeV: [PhysRevD.110.030001](https://arxiv.org/abs/110.030001)

P2: [arXiv:1802.04759](https://arxiv.org/abs/1802.04759)

SoLID: [arXiv:1302.6263](https://arxiv.org/abs/1302.6263)

MOLLER: [arXiv:1411.4088](https://arxiv.org/abs/1411.4088)

CMS: [arXiv:2408.07622](https://arxiv.org/abs/2408.07622)

LHC high mass: [arXiv:2302.10782](https://arxiv.org/abs/2302.10782)

Drell-Yan Forward-backward asymmetry (A_{FB}) (measures $\sin^2 \theta_{\text{eff}}^l$ at the Z pole)

- Key observable for mixing angle extraction

Asymmetry at the Z pole is sensitive to $\sin^2 \theta_{\text{eff}}^l$

θ : Angle btw incoming and outgoing particles in Collins-Soper frame.

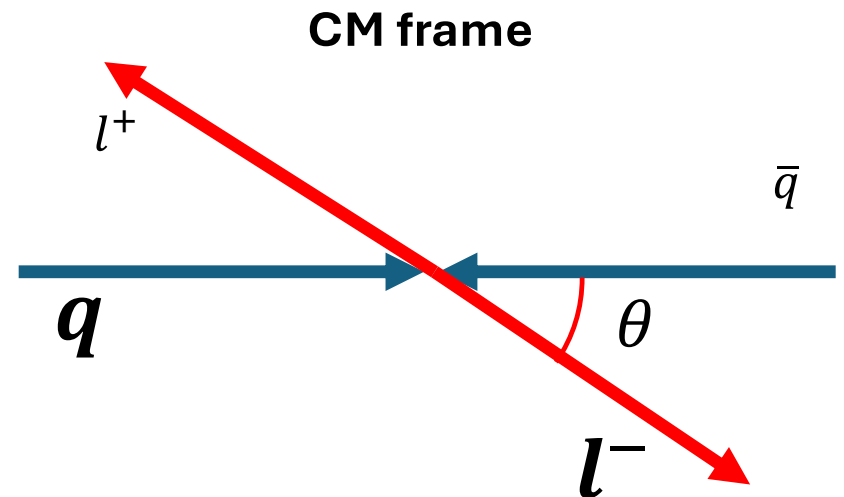
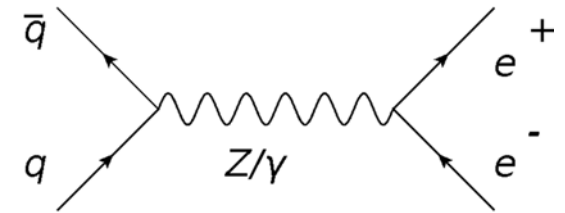
$\cos \theta > 0$, forward event (F)

$\cos \theta < 0$, backward event (B)

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B}$$

At the LHC The direction of the quark is defined as the direction of the rapidity of the dilepton.

The dilution from events for which this is incorrect is derived from [Parton Distribution Functions \(PDFs\)](#)



A precise measurement of $\sin^2 \theta_{eff}^l$ at hadron collider has been done at *CMS* using the following techniques

Angular event weighting method for A_{FB} analyses: A. Bodek. Eur. Phys. J. C67 (2010) 321

Systematic errors in acceptance & efficiency mostly cancel:

The method yields the full acceptance experimental A_{FB} (or A_4), but includes the effects final state photon radiation (FSR) and detector resolution smearing. used in CDF (Tevatron) and CMS (LHC)

Precise lepton momentum/energy scale (and modeling resolution) A. Bodek et al. , Eur. Phys. J. C72 (2012) 2194. used in CDF (Tevatron) and CMS (LHC 8 and 13 TeV)

(first extracted for muons and then updated for both muons and electrons and forward electrons (HF) in Run 2 13 TeV analysis)

Most important” In situ reduction of PDF errors by PDF reweighting/profling A.Bodek et al , Eur. Phys. H. (2016) C76 . **Using mass dependence of A_{FB} (and also W_{asy} , and W/Z cross section ratio)**

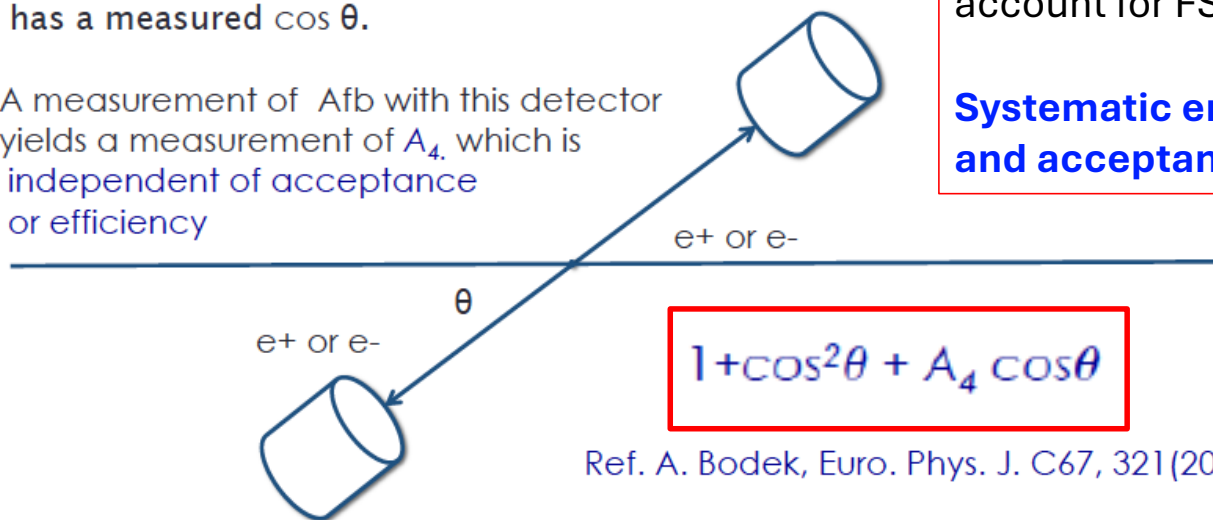
In situ cancellation of experimental systematic errors

Angular weighted A_{FB} is the same as A_{FB} for full acceptance (i.e. A_4) but smeared by experimental resolution and final state radiation, FSR

A. Bodek, Eur. Phys. J. C67 (2010) 321

Imagine a detector with acceptance for only one value of $\cos \theta$. Each event has a measured $\cos \theta$.

A measurement of A_{FB} with this detector yields a measurement of A_4 , which is independent of acceptance or efficiency



Do the same in data and MC to account for FSR and resolution.

Systematic errors in efficiencies and acceptance mostly cancel.

$$1 + \cos^2 \theta + A_4 \cos \theta$$

Ref. A. Bodek, Euro. Phys. J. C67, 321(2010)

$\cos \theta = 1$ yields best measurement of A_4 . $\cos \theta = 0$ yields no measurement of A_4

We can combine measurements of A_4 with different detectors at different $\cos \theta$ by weighting events. Events with $\cos \theta = 0$ have zero weight.

Events with $\cos \theta = 1$ have maximum weight. $\rightarrow$ obtain smaller statistical error.

$A_{FB}(\text{all } \cos \theta) = (3/8) A_4 \rightarrow$ No acceptance corrections needed.

Profiling with mass dependence of the Forward-backward asymmetry (A_{FB}) to constrain PDFs

A. Bodek et al., Eur. Phys. J. (2016) C76

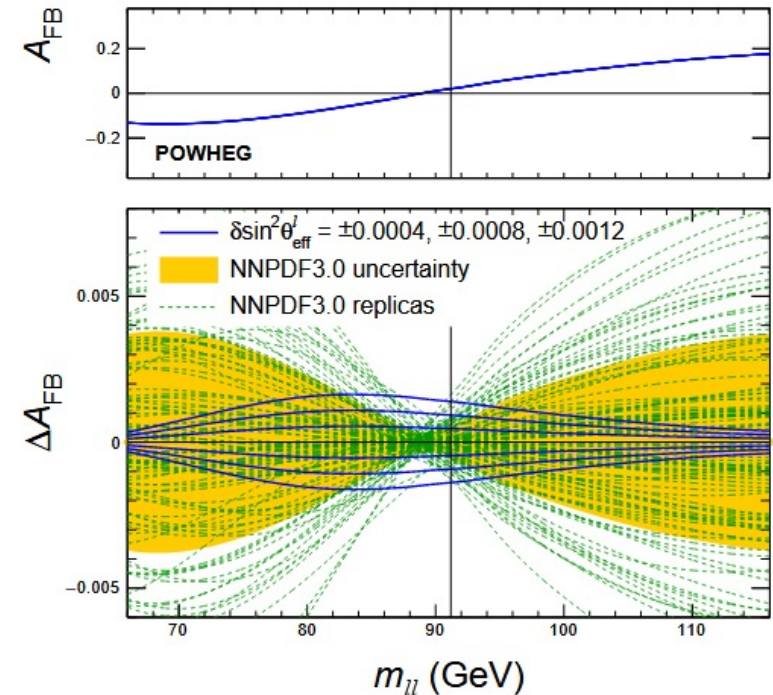
$$\frac{d\sigma_{DY}}{d\cos\theta} \propto (1 + \cos^2\theta) + \frac{A_0}{2}(1 - 3\cos^2\theta) + A_4\cos\theta$$

- $A_{FB} = \frac{3}{8}A_4$, directly related to the mixing of $U(1)$ and $SU(2)_L$

- A_{FB} at Z peak
 - Sensitive to $\sin^2\theta_{eff}^l$ and PDFs

- A_{FB} in sideband region
 - Small sensitivity to mixing angle
 - Large sensitive to PDFs

→ We use this to constrain PDFs
And reduce PDF errors (profiling)



$$\sin^2\theta_{eff}^l = 0.23152 \pm 0.00031$$

CMS 2025 Phys. Lett. B 866 (2025) 139526.

Afb profiling- Constraining PDF in situ- rapidity bins

These extracted PDF constraints are limited by the size of the statistical sample

The resulting PDFs uncertainties are can be reduced with larger statistical samples.

The errors in **Afb** are dominated by statistical error (since angular weighting is used).

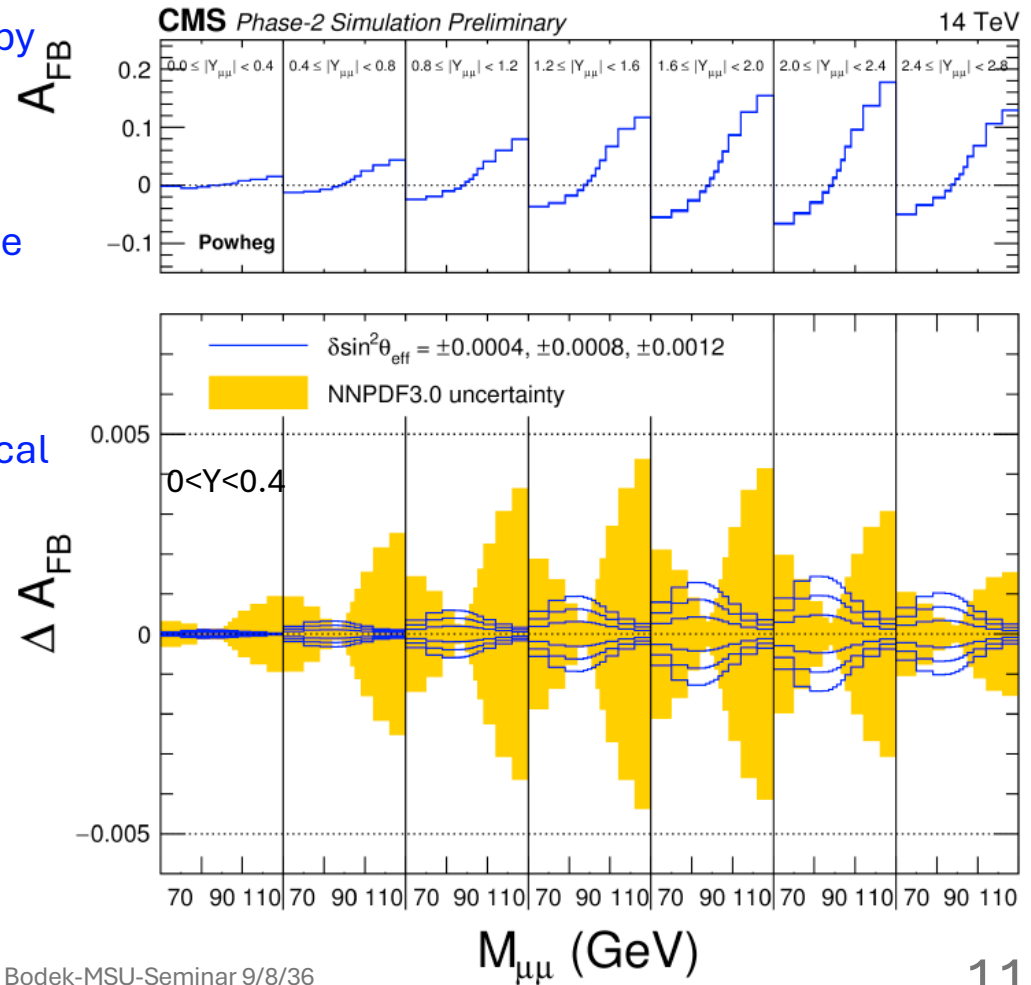
Before Profiling

$$\text{CT18Z } 0.23166 \pm 0.00056$$

After Profiling. CT18Z

$$\sin^2 \theta_{\text{eff}}^l = 0.23152 \pm 0.00031$$

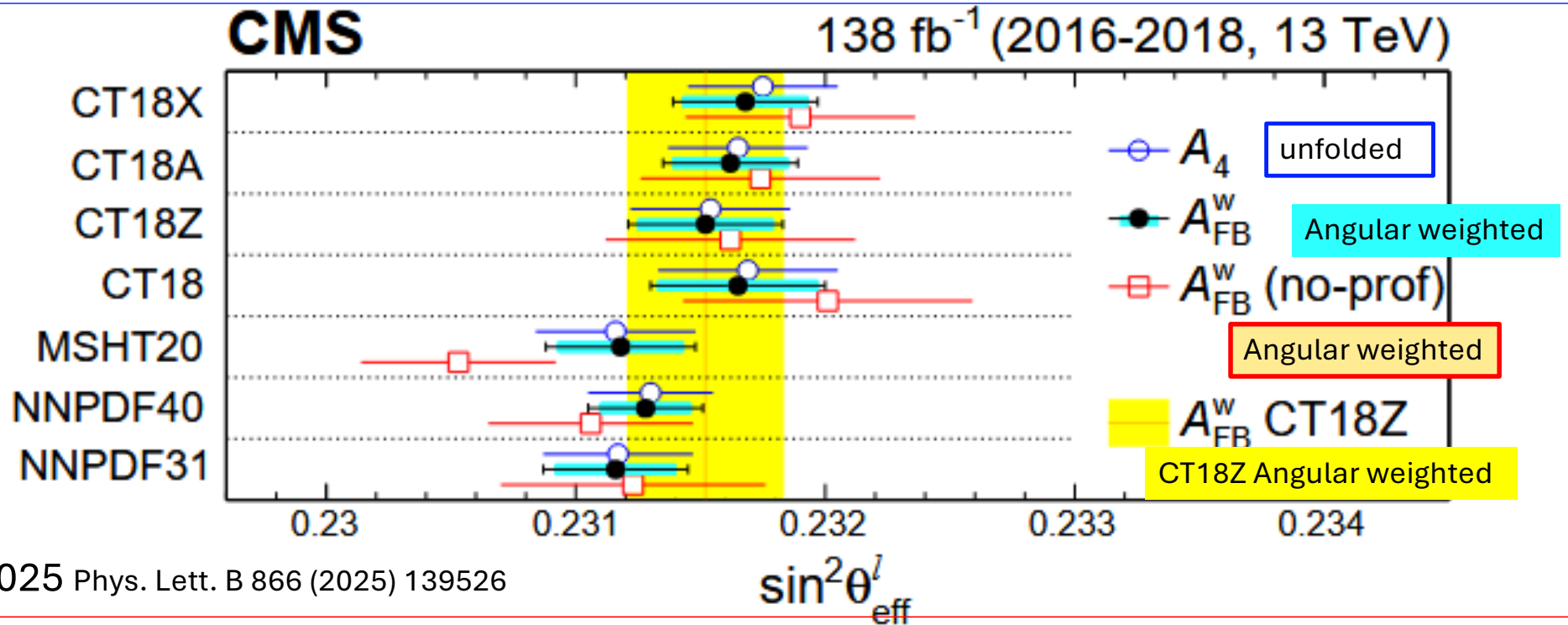
1/21/2026 Rochester NY



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$\sin^2 \theta_{\text{eff}}^l = 0.23166 \pm 0.00056$ (0.00010 *stat*, 0.00014 *syst*, 0.00009 *theory*, 0.00053 *PDFs*). **No profiling**

$\sin^2 \theta_{\text{eff}}^l = 0.23152 \pm 0.00031$ (0.00010 *stat*, 0.00014 *syst*, 0.00009 *theory*, 0.00027 *PDFs*). **After profiling**



CMS 2025 Phys. Lett. B 866 (2025) 139526

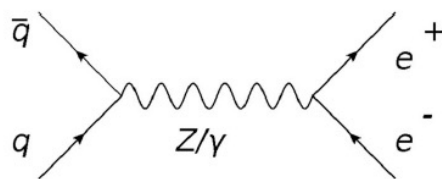
Issue: Some residual PDF-set dependance of CMS 13 TeV published $\sin^2 \theta_{\text{eff}}^l$ after Afb profiling (though much better than No Afb profiling)

7 PDF sets were investigated. (if PDFs were sufficiently constrained, all PDF results should be within 1 σ)

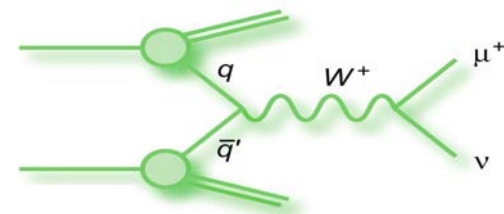
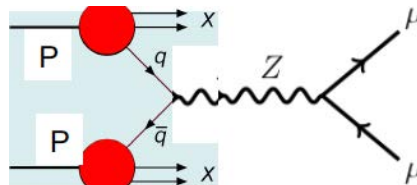
Reduce PDF errors by PDF reweighting/profiling **with additional data**
(only use asymmetry and ratio measurements)

A.Bodek et al , Eur. Phys. H. **(2016)** C76 (proposed constraints on parton distribution functions using both: *mass dependence of A_{FB} and W asymmetry*)

- **Mass dependence of A_{FB}** (Constrains mostly u-ubar and d-dbar)
mass dependent A_{FB} measured in the same detector at the same time.
(used in mixing angle analyses CDF 2026, CMS 8 TeV 2018, CMS 13 TeV 2024)
- **W^+/W^- asymmetry.** (Further constrains u-dbar and d-ubar) –
 W produced via u-dbar, d-ubar, s-cbar, c-sbar. not used in CMS 2024 analysis – This talk
- (can also Add) **W/Z cross section ratio** (constrains strange quarks)
 Z produced via u-ubar, d-dbar, s-sbar, c-cbar. not used in CMS 2024 analysis—This talk

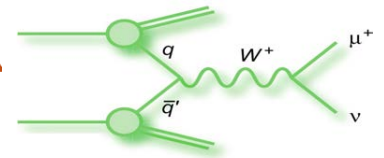


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Improved analysis of $\sin^2 \theta_{eff}^l$ with unfolded CMS 13 TeV Afb. [arXiv:2508.18022](https://arxiv.org/abs/2508.18022)

- Start with **published CMS unfolded A4** (M,y) at 13 TeV profiled with Afb (A4) (initial constraints on u quarks and d quarks)
- Add new high statistics **published CMS W decay lepton asymmetry** data at 13 TeV- (further constrains d quarks)
- Add (precise **published CMS W/Z** cross section ratios at 5 and 13 TeV (further constrains s quarks)
- **Perform a combined profiling analysis of all three data sets with additional (total of 19) modern PDF sets.**
- **Only asymmetry and ratio measurements are used (small systematic errors)**
- **Use open-source fit platform xFitter.**



2025: *Improved analysis*: Investigate 19 PDF sets

NNPDF40 (8)

Neural network base
Asymmetric strange

NNPDF40_nnlo *

NNPDF40_nnlo_qed

NNPDF40_nnlo_mhou

NNPDF40_nnlo_qed_mhou

NNPDF40_an3lo

NNPDF40_an3lo_qed

NNPDF40_an3lo_mhou

NNPDF40_an3lo_qed_mhou

MSHT20 (4)

Polynomial parameterization
Asymmetric strange

MSHT20nnlo_as118

MSHT20qed_nnlo

MSHT20an3lo_as118 *

MSHT20qed_an3lo

CT18 (6)

Polynomial parameterization
Symmetric strange

CT18XNNLO: alternative scale choice

CT18ANNLO: including ATLAS 7 TeV W/Z measurement

CT18ZNNLO: A+X (Nominal PDF) *

CT18qed_proton

CT18As_LatNNLO: asymmetric strange

PDF4LHC21 (1)

Combination of CT18+MSHT20+NNPDF31
Asymmetric strange

PDF4LHC21_40

“qed”: photon PDF and QED evolution

“an3lo”: approximate N3LO QCD

“mhou”: missing higher order uncertainty

* Yielded best post profiling χ^2

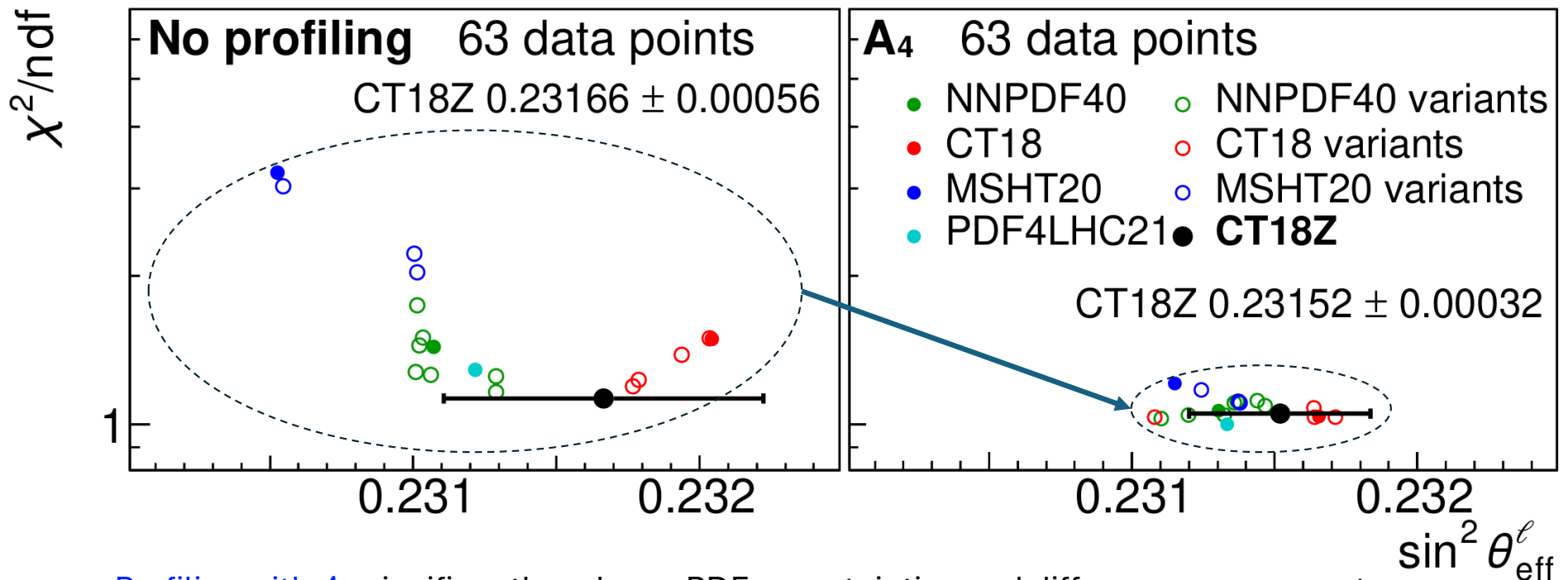
Extraction of $\sin^2 \theta_{\text{eff}}^l$

- Minimizing χ^2

$$\chi^2(s, \beta_{\text{exp}}, \beta_{\text{th}}) = \sum_{i=1}^{N_{\text{data}}} \frac{(\sigma_i^{\text{exp}} + \sum_j \Gamma_{ij}^{\text{exp}} \beta_{j,\text{exp}} - \sigma_i^{\text{th}}(s) - \sum_k \Gamma_{ik}^{\text{th}} \beta_{k,\text{th}})^2}{\Delta_i^2} + \sum_j \beta_{j,\text{exp}}^2 + \sum_k \beta_{k,\text{th}}^2$$

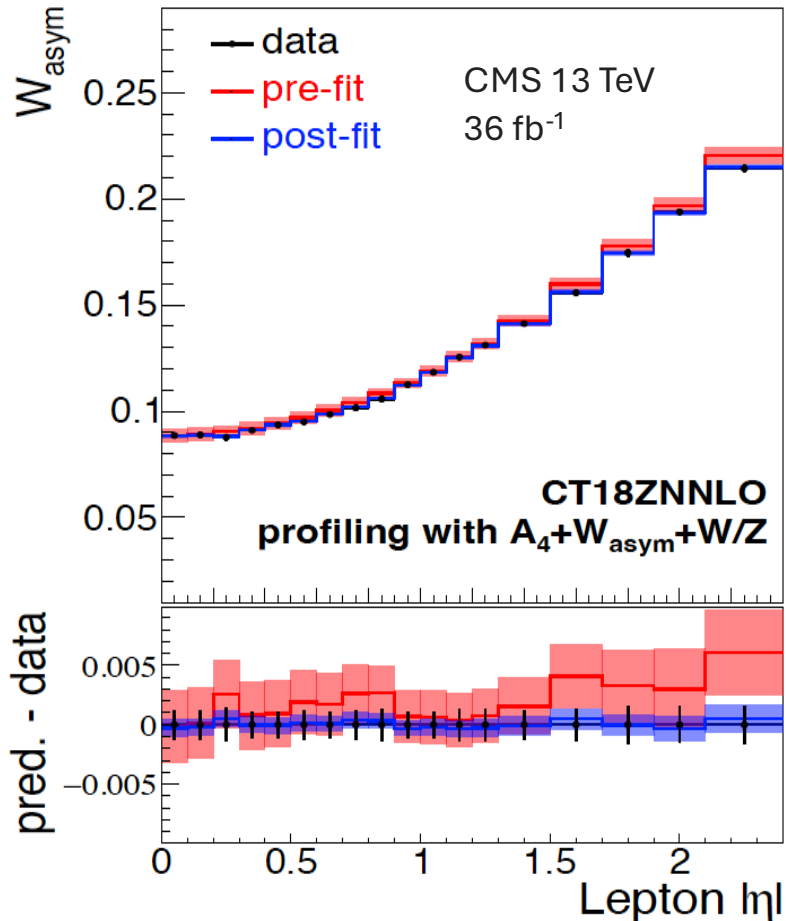
- $\mathbf{s}$: $\sin^2 \theta_{\text{eff}}^l$
- σ_i^{exp} and $\sigma_i^{\text{th}}(\mathbf{s})$: measurement and prediction of i -th data point
- Δ_i : uncorrelated uncertainty of i -th data point
- $\beta_{j,\text{exp}}$ and Γ_{ij}^{exp} : experimental nuisance parameters and their impact matrixes
 - decomposition of A_4 covariance matrix
- $\beta_{j,\text{th}}$ and Γ_{ij}^{th} : theory nuisance parameters and their impact matrixes
 - PDF hessian error
 - electroweak theory uncertainty
- QCD scale uncertainty is evaluated outside the fit using the standard 7-point variation
 - Repeat the fit for each scale choice and take the envelope

19 PDF sets: $\sin^2 \theta_{\text{eff}}^l$ with CMS 13 TeV $A_4(\text{M},\text{y})$ profiling only



- Profiling with A_4 significantly reduces PDF uncertainties and differences across sets
- CT18 family tends to yield a bit larger $\sin^2 \theta_{\text{eff}}^l$ (consistent with the original CMS paper)

CMS 13 TeV W-decay lepton charge asymmetry (W_{asym}) further constrains d quarks

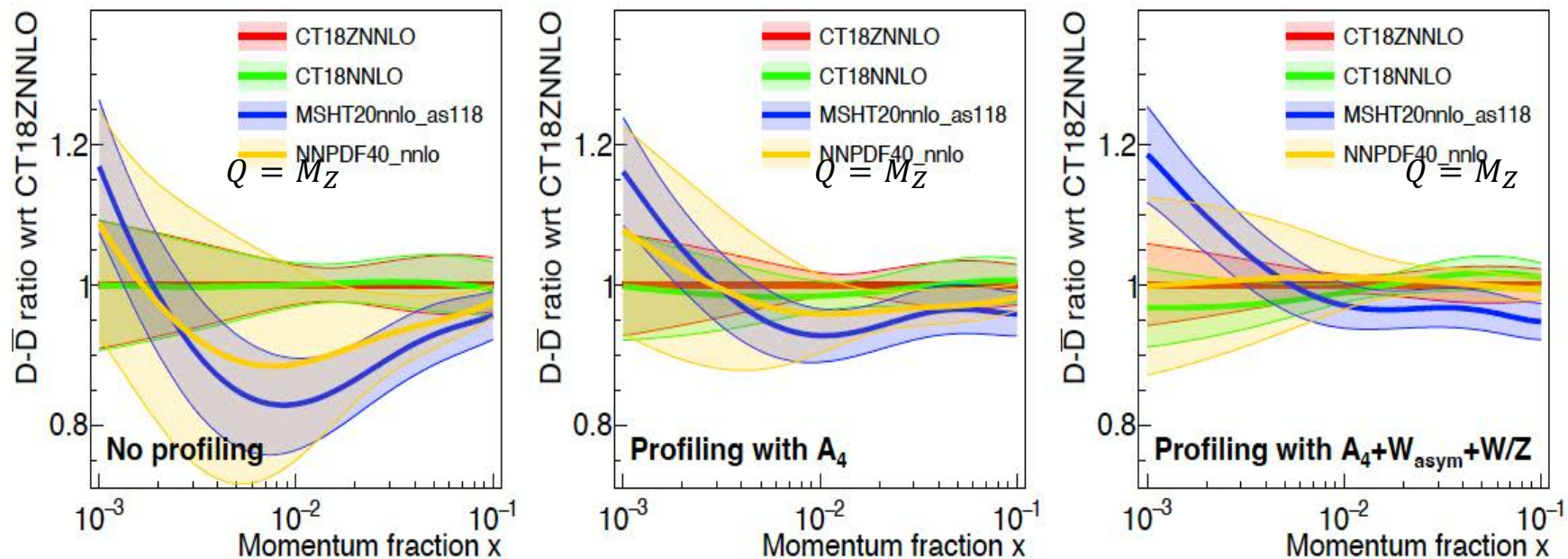


- W asymmetry can help constrain d/u PDF and reduce uncertainty of $\sin^2 \theta_{\text{eff}}^l$ (as investigated in [Eur. Phys. J. C76 \(2016\) 115](#))
- CMS W charge asymmetry measurements at **13 TeV**
 - [Phys. Rev. D 102 \(2020\) 092012](#)
 - 18 data points in bins of lepton pseudorapidity $|\eta_l|$

$$W_{\text{asym}}(|\eta_l|) = \frac{\sigma_{W^+}(|\eta_l|) - \sigma_{W^-}(|\eta_l|)}{\sigma_{W^+}(|\eta_l|) + \sigma_{W^-}(|\eta_l|)}$$

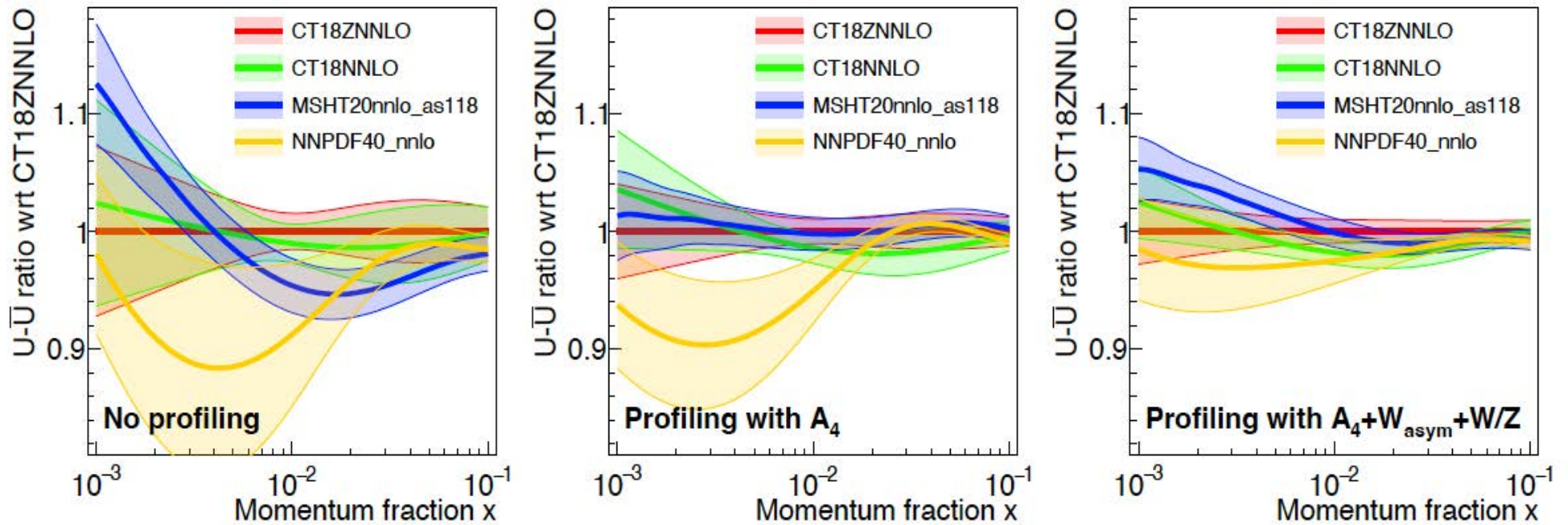
- Prediction from MCFM (NNLO)

Impact of ($W_{\text{asym}} + W/Z$) on down-type quarks. (analysis x range 0.01-0.1)



- Examine $D - \bar{D} \equiv (d + s + b) - (\bar{d} + \bar{s} + \bar{b})$, rather than $d_v = d - \bar{d}$
 - For fair comparison across PDFs with different assumptions on (a)symmetric sea-quark momentum distributions
- Adding W_{asym} further constrains the down-type quarks (strange constrained by W/Z)

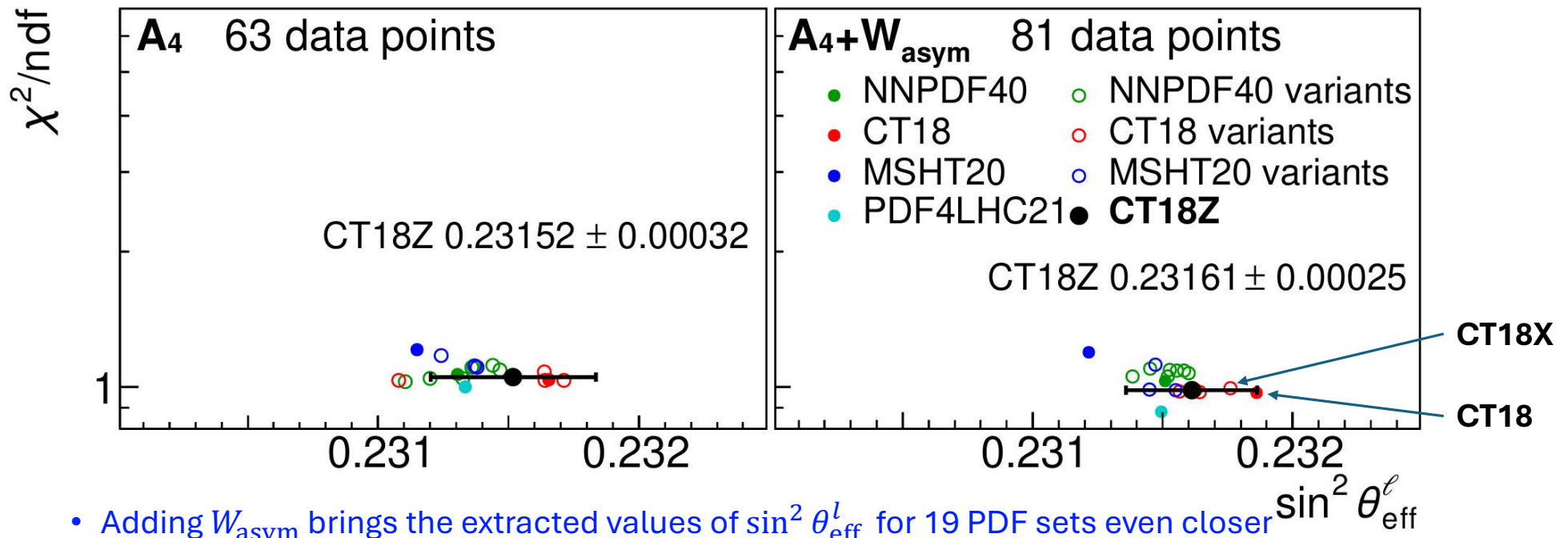
Impact of ($W_{\text{asym}} + W/Z$) on up-type quarks



- Examine $U - \bar{U} \equiv (u + c) - (\bar{u} + \bar{c})$ $u_v = u - \bar{u}$
- Adding W_{asym} also further constrains the up-type quarks

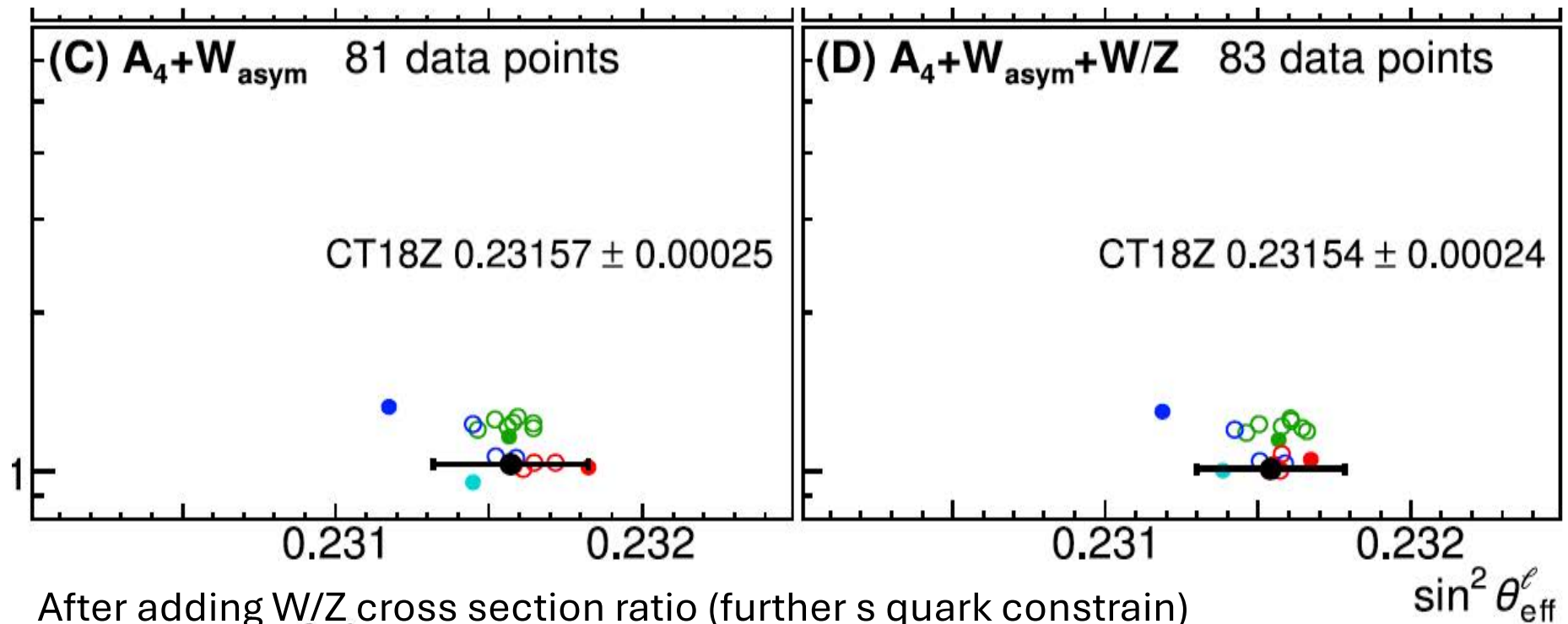
19 PDF sets: $\sin^2 \theta_{\text{eff}}^l$ with combined $A_4 + W_{\text{asym}}$ profiling

Before profiling CT18Z 0.23166 ± 0.00056



- Adding W_{asym} brings the extracted values of $\sin^2 \theta_{\text{eff}}^l$ for 19 PDF sets even closer
 - MSHT20 shows a lower $\sin^2 \theta_{\text{eff}}^l$ than all other MSHT variants
 - Two of CT18 family (CT18 and CT18X) are within 1σ band, but gives slightly higher values
- CT18 and CT18X PDFs have smaller strange-quark distributions $\rightarrow$ higher $\sin \theta_{\text{eff}}^l$

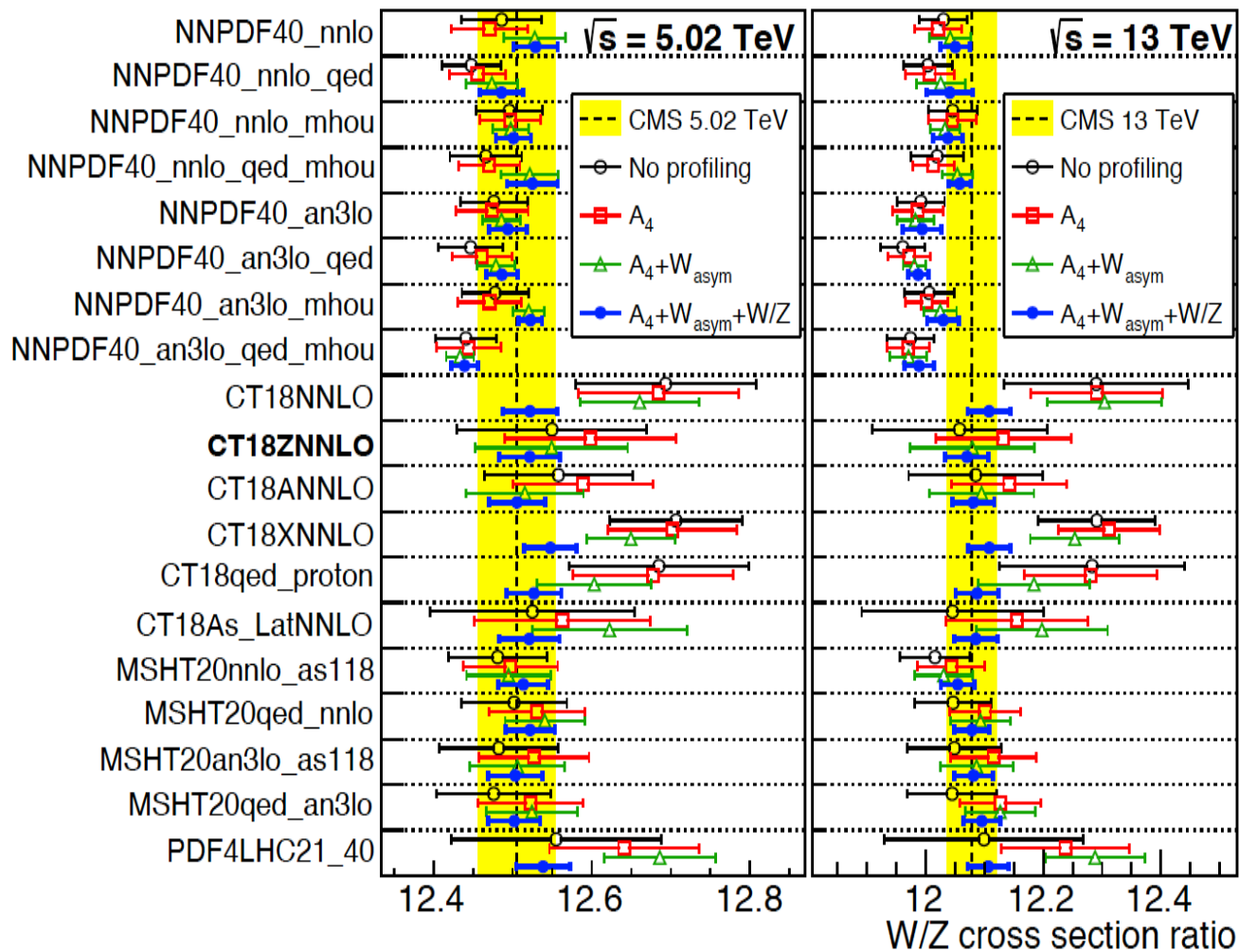
19 PDF sets: $\sin^2 \theta_{\text{eff}}^l$ with W/Z profiling (further constrains s quarks)



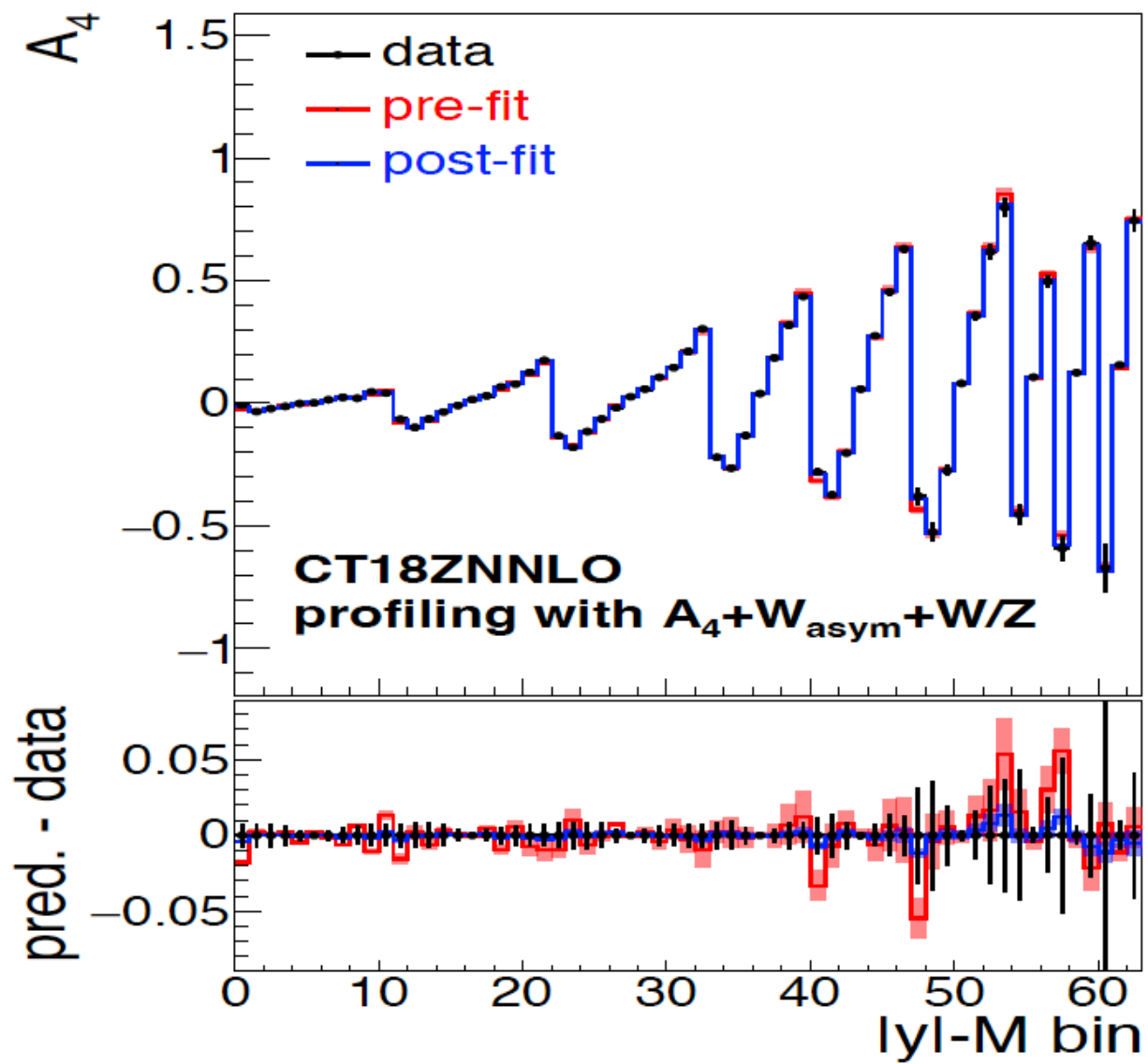
After adding W/Z cross section ratio (further s quark constrain)

- CT18 and CT18X get closer to CT18Z
- Only one MSHT20 PDF remains outside 1σ band

W/Z cross section ratio constrains s-quark. (Blue points)

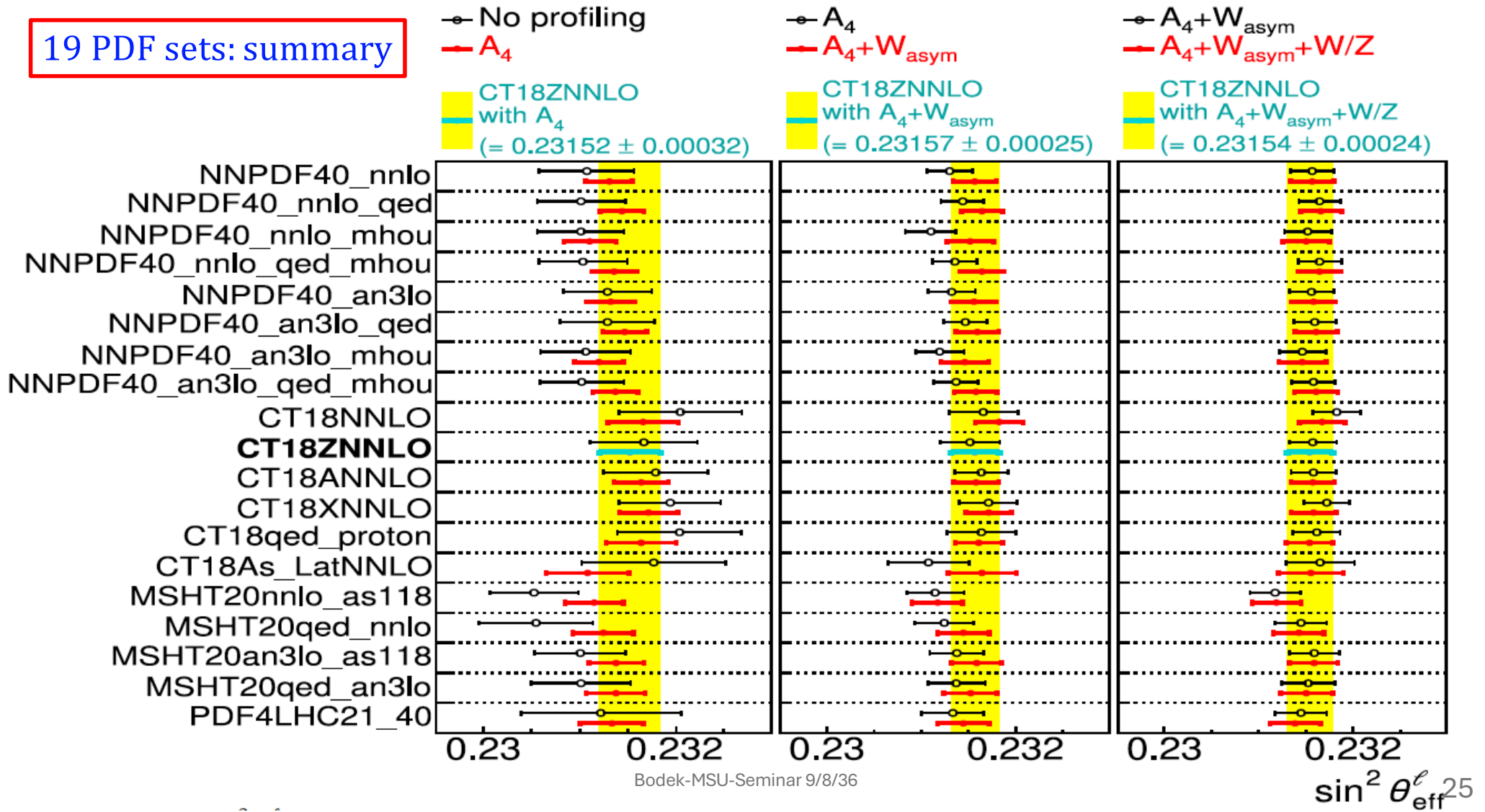


- The W/Z ratio is strongly anti-correlated with the strange-quark content
- CMS W/Z ratio (2 data points)
 - [JHEP 04 \(2025\) 162](#)
 - $12.505 \pm 0.037_{\text{stat}} \pm 0.032_{\text{syst}}$ (5.02 TeV)
 - $12.078 \pm 0.028_{\text{stat}} \pm 0.032_{\text{syst}}$ (13 TeV)
- Prediction from DYTurbo (NNLO+NNLL)
- **Blue points are W/Z ratio predictions after profiling with all three data sets.**

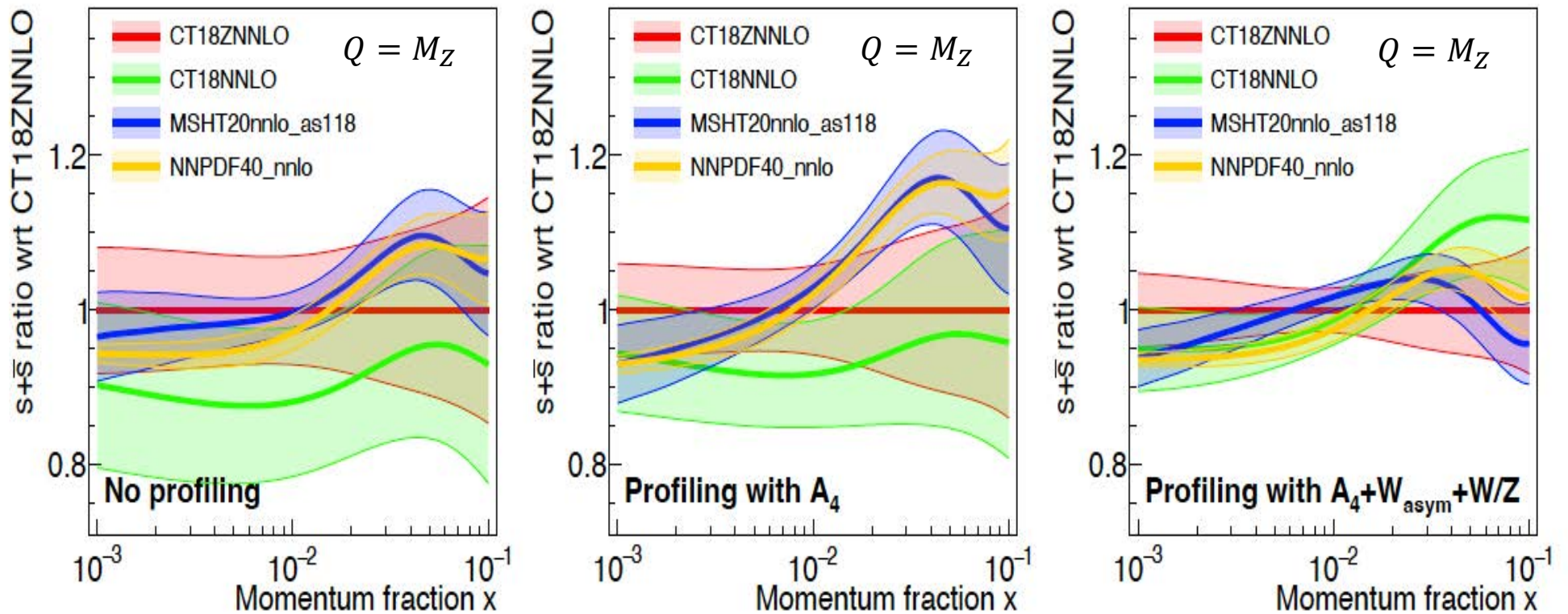


$$A_{\text{FB}}(\text{full acceptance}) = (3/8)A_4$$

19 PDF sets: summary

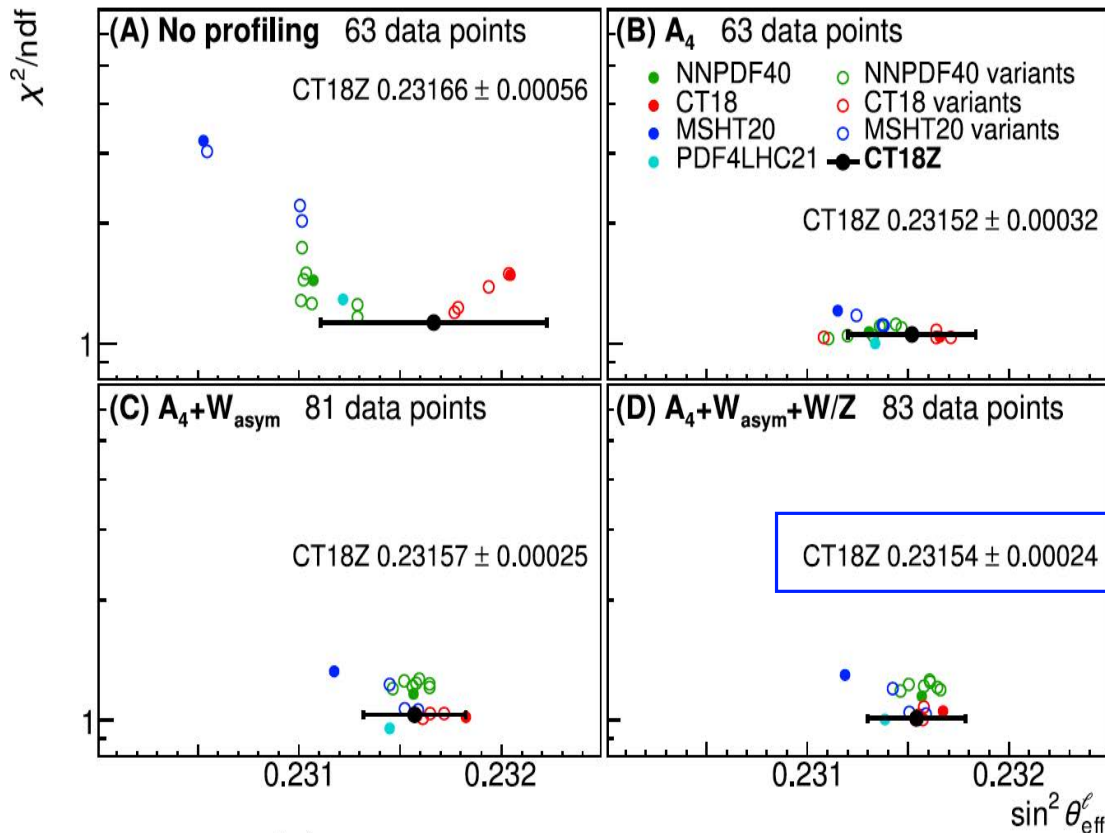


Impact of W/Z constraints on *strange quark distribution*
 Adding W/Z increase strange quark content of CT18



Summary results with:

No profiling vs (A_4) vs $(A_4 + W_{\text{asym}})$ vs $(A_4 + W_{\text{asym}} + W/Z)$ profiling



$$\sin^2 \theta_{\text{eff}}^l = 0.23154 \pm 0.00024 \text{ (CT18Z)}$$

- Most extracted $\sin \theta_{\text{eff}}^l$ values lie within CT18Z uncertainty after profiling with A_4 , W_{asym} , and the W/Z cross section ratio
 - Robust against 19 different PDF parameterizations and assumptions
- MSHT20 alone remains outside the CT18Z uncertainty band
 - a higher χ^2 may imply limited parametrization flexibility

The tolerance parameters are about 6, 4, and 1.6 , for the CTEQ, MSHT and NNPDF PDF sets , respectively

The latest CTEQ PDF sets were released in 2019, the latest MSHT PDF sets were released in 2020, and the latest NNPDF sets were released in 2021.

PDF	A_4 without profiling (63 data points)			A_4 profiling (63 data points)			$A_4 + W_{\text{asym}} + W/Z$ profiling (83 data points)		
	$\sin^2 \theta_{\text{eff}}^{\ell}$	Diff. from CT18Z	χ^2	$\sin^2 \theta_{\text{eff}}^{\ell}$	Diff. from CT18Z	χ^2	$\sin^2 \theta_{\text{eff}}^{\ell}$	Diff. from CT18Z	χ^2
NNPDF40									
nnlo_as_01180_hessian [15]	23107 ± 49	-59	89	23130 ± 24	-22	66	23157 ± 23	3	94
nnlo_as_01180_qed [16]	23102 ± 46	-64	108	23144 ± 23	-8	69	23166 ± 22	12	97
nnlo_as_01180_mhou [17]	23101 ± 45	-65	79	23110 ± 27	-42	64	23150 ± 24	-4	101
nnlo_as_01180_qed_mhou [16]	23103 ± 46	-63	93	23136 ± 24	-16	69	23164 ± 23	10	99
an3lo_as_01180 [18]	23129 ± 46	-37	72	23132 ± 25	-20	65	23158 ± 23	4	100
an3lo_as_01180_qed [18]	23129 ± 49	-37	78	23147 ± 23	-5	68	23160 ± 22	6	103
an3lo_as_01180_mhou [18]	23106 ± 47	-60	78	23120 ± 26	-32	65	23146 ± 25	-8	97
an3lo_as_01180_qed_mhou [18]	23102 ± 44	-64	90	23137 ± 23	-15	69	23161 ± 23	7	102
CTEQ									
CT18NNLO [11]	23204 ± 64	38	92	23166 ± 36	14	64	23167 ± 24	13	86
CT18Znnlo [11]	23166 ± 56	0	70	23152 ± 32	0	65	23154 ± 24	0	83
CT18ANNLO [11]	23179 ± 54	13	76	23164 ± 28	12	67	23157 ± 23	3	82
CT18XNNLO [11]	23194 ± 53	28	86	23171 ± 30	19	64	23158 ± 24	4	88
CT18QED-PROTON [12]	23204 ± 64	38	93	23164 ± 36	12	64	23154 ± 24	0	82
CT18As_LATNNLO [13]	23177 ± 75	11	74	23108 ± 43	-44	64	23155 ± 35	1	84
MSHT20									
MSHT20nnlo_as118 [20]	23053 ± 46	-113	200	23115 ± 30	-37	75	23119 ± 26	-35	106
MSHT20qed_nnlo [21]	23055 ± 59	-111	188	23124 ± 31	-28	73	23142 ± 27	-12	98
MSHT20an3lo_as118 [22]	23101 ± 47	-65	138	23138 ± 29	-14	69	23159 ± 26	5	85
MSHT20qed_an3lo [23]	23101 ± 52	-65	126	23137 ± 31	-15	69	23151 ± 28	-3	86
PDF4LHC21_40 [37]	23122 ± 83	-44	80	23133 ± 33	-19	62	23139 ± 27	-15	82

NNPDF40, T=1.6
Lowest χ^2
Among
NNPDF 4.0
variants
Error 49 -> 23

CT18Z, T=6
Lowest χ^2
Among CT18
Variants
Error 56 -> 24

MRST20an3lo, T=4
Lowest χ^2
Among MRST20
variants
Error 47 -> 26

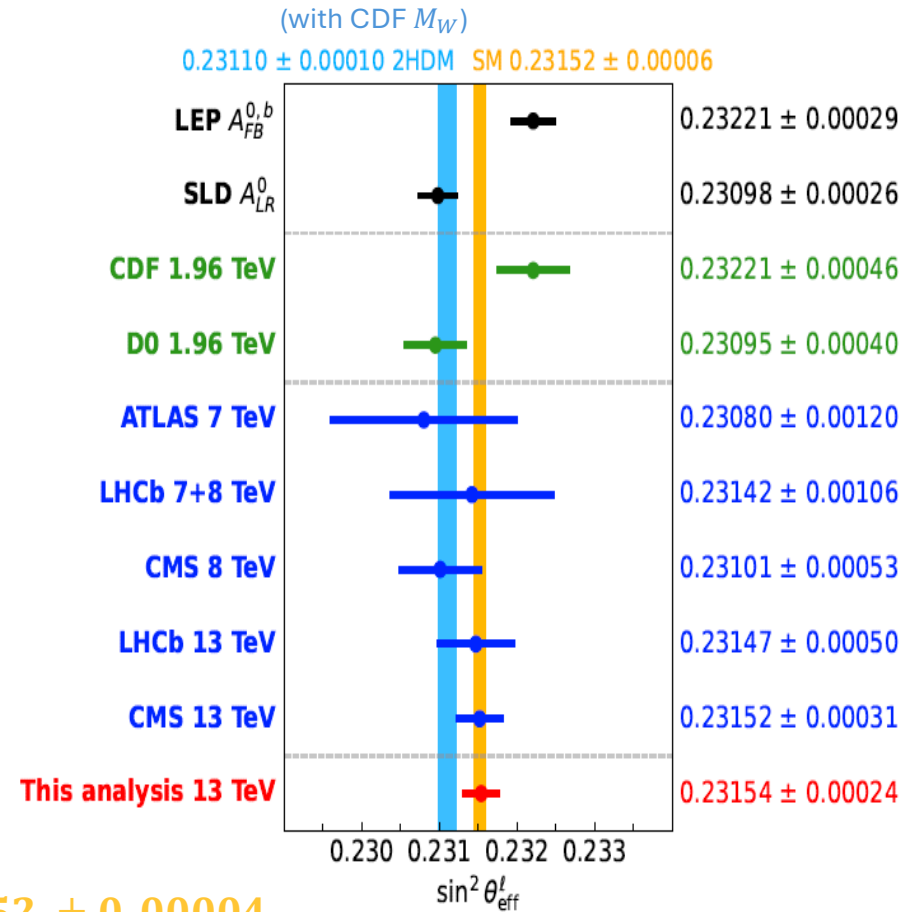
Comparison to theory and other measurements

$$\sin^2 \theta_{\text{eff}}^l = 0.23156 \pm 0.00024$$

- Improved extraction of $\sin^2 \theta_{\text{eff}}^l$ from CMS unfolded A_4 measurements at 13 TeV
 - with additional PDF constraints from W_{asym} and W/Z cross section ratio
 - Reduced Uncertainty
- Competitive with SLD error of 0.00026

0.00031 (CMS 13 TeV) $\rightarrow$ 0.00024 (This analysis)

$$\sin^2 \theta_{\text{eff}}^l = 0.23154 \pm 0.00024 \quad \text{Vs SM} \quad 0.23152 \pm 0.00004$$



The tolerance parameters are about 6, 4, and 1.6 , for the CTEQ, MSHT and NNPDF PDF sets , respectively

The latest CTEQ PDF sets were released in 2019, the latest MSHT PDF sets were released in 2020, and the latest NNPDF sets were released in 2021.

PDF	A_4 without profiling (63 data points)			A_4 profiling (63 data points)			$A_4 + W_{\text{asym}} + W/Z$ profiling (83 data points)		
	$\sin^2 \theta_{\text{eff}}^{\ell}$	Diff. from CT18Z	χ^2	$\sin^2 \theta_{\text{eff}}^{\ell}$	Diff. from CT18Z	χ^2	$\sin^2 \theta_{\text{eff}}^{\ell}$	Diff. from CT18Z	χ^2
NNPDF40									
nnlo_as_01180_hessian [15]	23107 ± 49	-59	89	23130 ± 24	-22	66	23157 ± 23	3	94
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nnlo_as_01180_qed_mhou [16]	23103 ± 46	-63	93	23136 ± 24	-16	69	23164 ± 23	10	99
an3lo_as_01180 [18]	23129 ± 46	-37	72	23132 ± 25	-20	65	23158 ± 23	4	100
an3lo_as_01180_qed [18]	23129 ± 49	-37	78	23147 ± 23	-5	68	23160 ± 22	6	103
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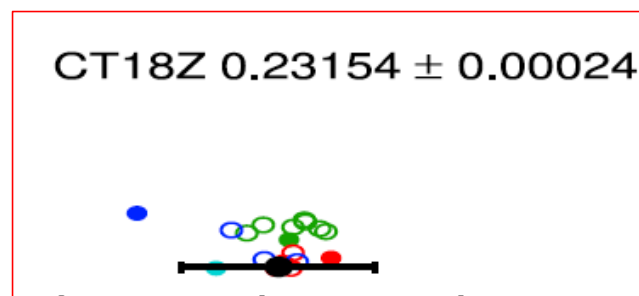
NNPDF40, T=1.6
Lowest χ^2
Among
NNPDF 4.0
variants
Error 49 -> 23

CT18Z, T=6
Lowest χ^2
Among CT18
Variants
Error 56 -> 24

MRST20an3lo, T=4
Lowest χ^2
Among MRST20
variants
Error 47 -> 26

Profiled PDF and Tolerance – back to mixing angle analysis

- The PDF constraints on mixing angle measurements using A_4 , W_{asym} , and W/Z at 13 TeV were done with $T^2 = 1$ (i.e. $\Delta\chi^2 = 1$) for 19 PDF sets.
- We found that the flexibility of 18 modern PDF parameterization is sufficient within this specific region of $Q \approx M_Z$ and $x = 0.001\text{--}0.1$ to describe **these three data sets (after profiling) with good χ^2** .
- Only one PDF set (NSHT20) could not accommodate the profiling (not understood)
- After profiling 18 PDFs are within half a standard deviation of CT18Z.



NNLO ODFs 83 data points
Lowest χ^2 After profiling

NNPDF4.0	23157 ± 23	$\chi^2=94$
CT18Z	23154 ± 24	$\chi^2=83$
MST20an3lo	23159 ± 26	$\chi^2=85$

“an3lo”: approximate N3LO QCD

Compare to Direct Measurements of the W mass

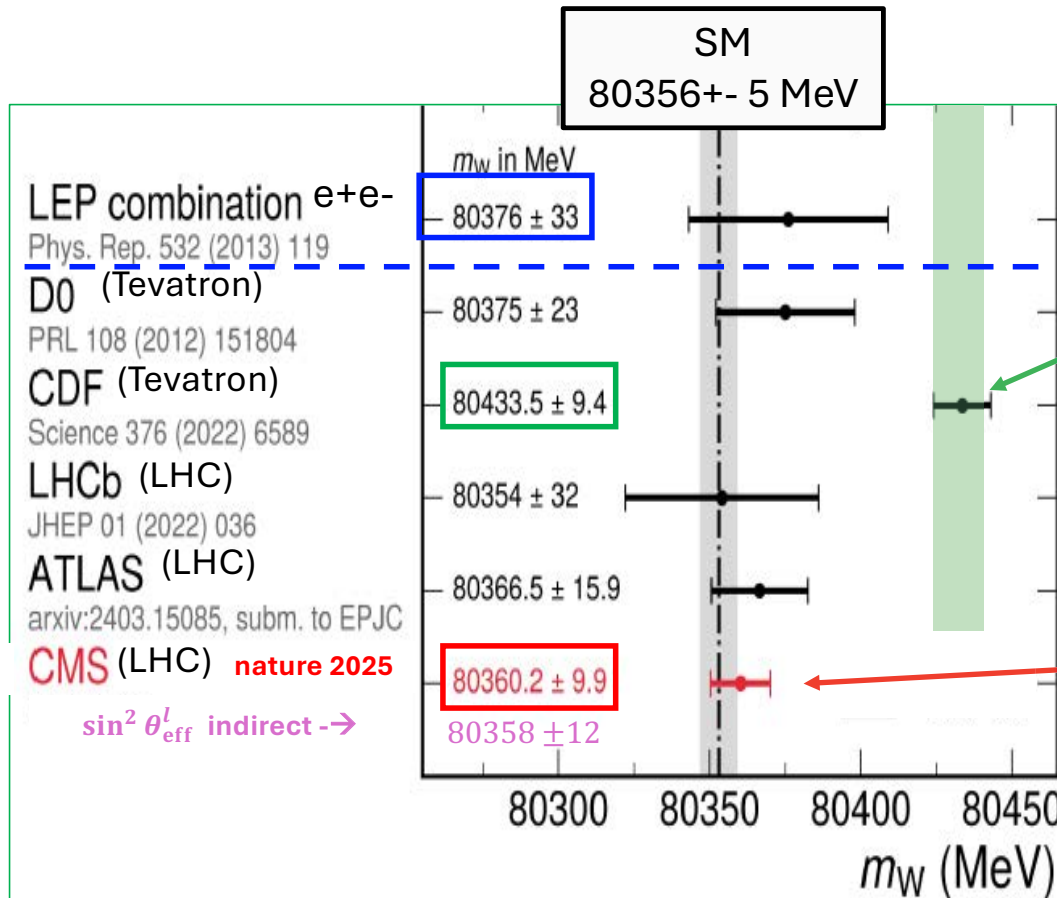
$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(M_Z^2, \sin^2 \theta_w)] \sin^2 \theta_w$$

$$\hookrightarrow \approx 1.0363$$

$$\sin^2 \theta_W (\text{on shell}) = 1 - \left(\frac{m_W}{m_Z} \right)^2$$

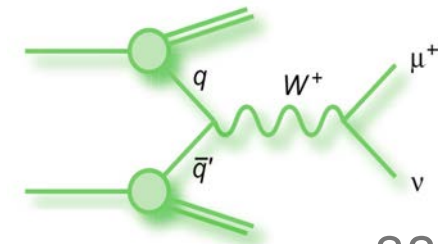
$$\sin^2 \theta_{\text{eff}}^l = 0.23156 \pm 0.00024$$

$$\rightarrow M_W(\text{indirect}) = 80358 \pm 12$$



CDF Tevatron σ 9.4 MeV
Disagreement with SM
2022

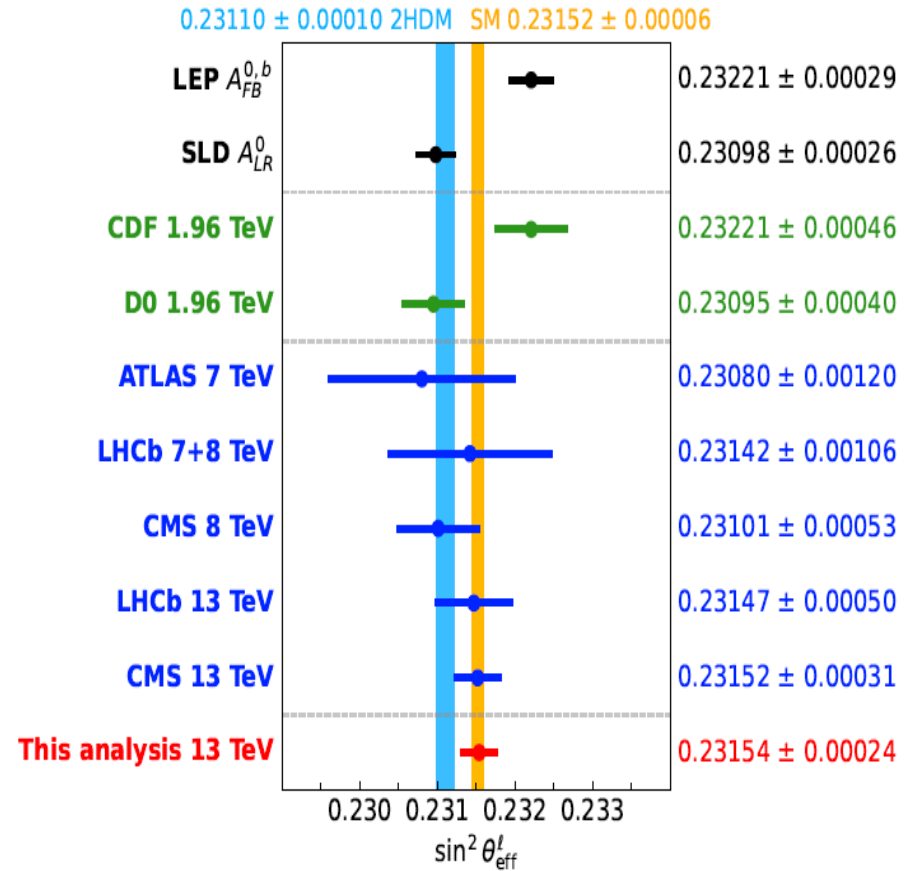
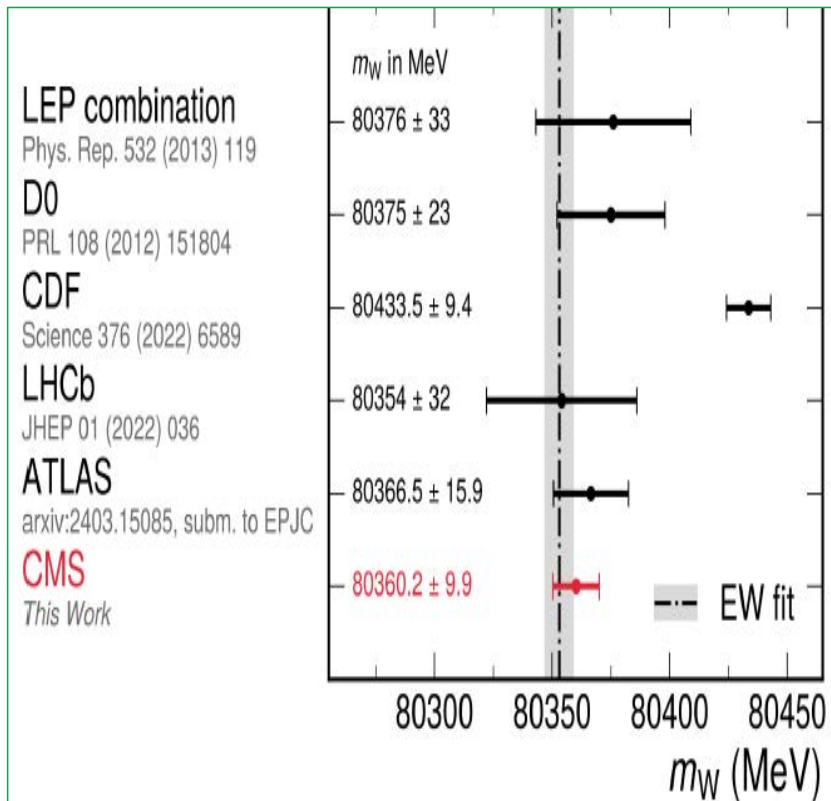
CMS at LHC σ 9.9 MeV
Agreement with SM 2024



[arXiv:2412.13872](https://arxiv.org/abs/2412.13872)

Summary of. W mass and mixing angle results

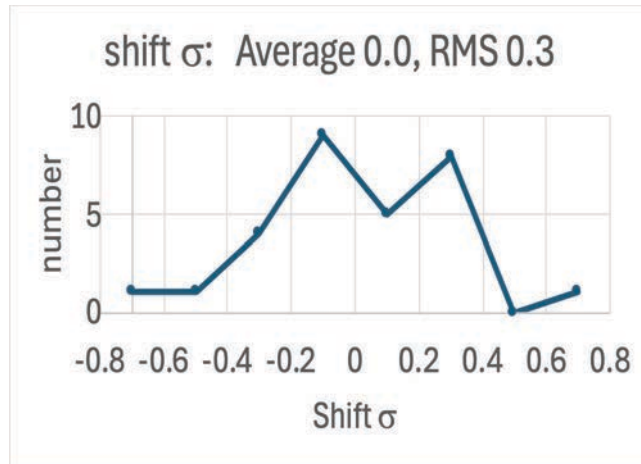
We are entering a new era of precision electroweak measurements at the LHC



Post profiling CT18Z. Eigenvalues shifts and errors (before profiling 0 and 1)

Post profiling shifts on the CT18Z average 0.009 with RMS 0.38.

Average uncertainty reduced from 1.0 to 0.883



Index	Shift	Uncertainty	Index	Shift	Uncertainty
1	-0.2698	0.8694	16	-0.0998	0.8569
2	0.4215	0.8383	17	0.4871	0.8999
3	0.7903	0.9111	18	-0.5412	0.9113
4	0.1190	0.9624	19	-0.1664	0.7318
5	0.3964	0.9353	20	-0.1035	0.9096
6	-0.1410	0.9035	21	0.2783	0.9340
7	0.1101	0.8725	22	0.3516	0.9429
8	0.1668	0.8608	23	-0.6438	0.8087
9	-0.3094	0.9057	24	0.1258	0.8168
10	0.4435	0.8992	25	0.1400	0.8849
11	0.3144	0.8146	26	-0.1593	0.8616
12	-0.2324	0.8963	27	-0.4221	0.9555
13	0.3711	0.8928	28	0.0709	0.8960
14	0.1572	0.8638	29	-0.8794	0.8651
15	-0.5092	0.9180	-	-	-

Table A.2. Shifts and uncertainties of the 29 nuisance parameters corresponding to the CT18Z set after profiling with A_4 , W_{asym} , and W/Z fiducial cross-section ratio. The prior uncertainties are defined to be 1.0, and before profiling the shifts are zero and the errors are 1.0 by definition.

C.-P. Yuan GLOBAL PDF fits



Tolerance for defining the allowed PDF errors



Source of PDF uncertainty

PDFs receive uncertainty from...

- Experimental uncertainty; statistic and systematic.
- Uncertainties from theoretical input, such like α_s , m_q , α .
- QCD and QED approximation in perturbation (or scale uncertainty).
- Numerical uncertainty from MC integration.
- Choice of the parametrization form of PDF at Q_0 scale.
- Methods for uncertainty estimation.

- Possible tension between different experimental data sets.

Typical Tolerance (the amount of $\Delta\chi^2$ from the central set) adopted by CT, and MSHT groups to define PDF errors of the Hessian sets, due to **different methodology**, at the 1σ level, are:

T^2 30 – 37 for CT
15 – 20 for MSHT

For NNPDF, with Monte Carlo sets, the effective Tolerance is about $2 - T^2$
→ Smaller PDF errors.

July 6 -16, 2022 @ Univ. of Pittsburgh

2022 CTEQ Summer School

Profiled PDF and Tolerance- Tolerance of 1.0 is used in mixing angle and W mass analyses

This procedure assumes that the PDF parametrization is sufficiently flexible in the relevant kinematic region, $Q \simeq M_Z$ and $x = 0.001-0.1$. The profiling is restricted to asymmetry and cross-section ratio measurements, whose uncertainties are statistically dominated.

At 13 TeV CMS very precise Drell-Yan Afb, W decay lepton charge asymmetry and W/Z cross section ratio. These precise data provide very strong constraints on PDFs in the W and Z kinematic region.

This ensures that the PDFs are consistent with the W and Z data that we use in our analysis.

It may be reasonable if needed to apply tolerance factors to systematic errors, since systematic errors are not as well understood as statistical errors.

There are two major differences between our analysis and Global PDF analyses,

- (1) Kinematic region limited to W and Z region
- (2) The data used is dominated by statistical errors which are well understood.

Profiled PDF and Tolerance- Should global PDFs fits be done differently?

The CMS mixing angle and W mass analysis at LHC use profiling with $\Delta\chi^2 = 1$. PDFs are constrained by (1)Angular weighted Afb above and below Z region. (2) W Charge Asym (3) W/Z ratio at 13 TeV. These errors are statistically dominated and the analysis is limited to W and Z boson kinematics. *Analysis is limited to the W and Z boson kinematic region.*

Global PDF fits

- Cover a wide range of Q (GeV~TeV) and x (0.0001~1). (vs $Q=M_Z$, 0.001-0.1 for mixing angle analysis)
- **Tolerance factor $T(\geq 1)$** accounts for **limited flexibility** of parameterization (to cover a large kinematic range) and **data inconsistencies (i.e. larger systematic errors)**

Consider using Precise Ratio and Asymmetry measurements as **candles or anchors** to ensure that

- (1) The parameterizations are able to accommodate these precise candles.
- (2) Discover unaccounted for systematic errors in other experiments.
- (3) If possible, tolerance factors should be applied to systematic errors only

Proposal: In the PDF fits, the precise candles should be given larger weights

–An historical example in the next page.

A Bodek: Nuclear Physics B (Proceedings Supplements), Vol. 139, p. 165-192 (2005)

Historical Example. How the re-analysis of all of SLAC provided structure functions that are used as candle for the higher energy muon experiments at CERM

1986 The Rochester group ran SLAC electron scattering experiment E140. We measured precise e-p, e-d cross sections and took data to cross normalize to all other SLAC experiment.

We reanalyzed all of the previous SLAC data with common Bardin radiative correction for all data. **We then did a global analysis of all SLAC experiment and cross normalized all experiment to SLAC E140**

We extracted F_2 and $R=\sigma_L/\sigma_T$ Global analysis normalization error 2%.

Phys.Lett.B 282 (1992) 475-482. Phys.Lett.B 250 (1990) 193-198

All 8 GeV and 20 GeV spectrometer experiments normalization are within 2% .
E89 at 60 degree used the 1.6 GeV spectrometer experiment normalization 5%.

Final normalizations are all within normalization uncertainties. The extracted structure functions errors are ~ 2%. These are low energy anchors

Normalization factors

Hydrogen

E49a	1.012
E49b	.981
E61	1.011
E87	.982
E89a †	.989
E89b	.953

Deuterium

E49a	1.001
E49b	.981
E61	1.033
E87	.986
E89a †	.985
E89b	.949
E139	1.008
E140	1.000

Physics Letters B 282 (1992) 475–482
North-Holland

**Precise measurements
of the proton and deuteron structure functions
from a global analysis
of the SLAC deep inelastic electron scattering cross sections ☆**

L.W. Whitlow, E.M. Riordan ¹, S. Dasu

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA

S. Rock

American University, Washington, DC 20016, USA

and

A. Bodek

University of Rochester, Rochester, NY 14627, USA

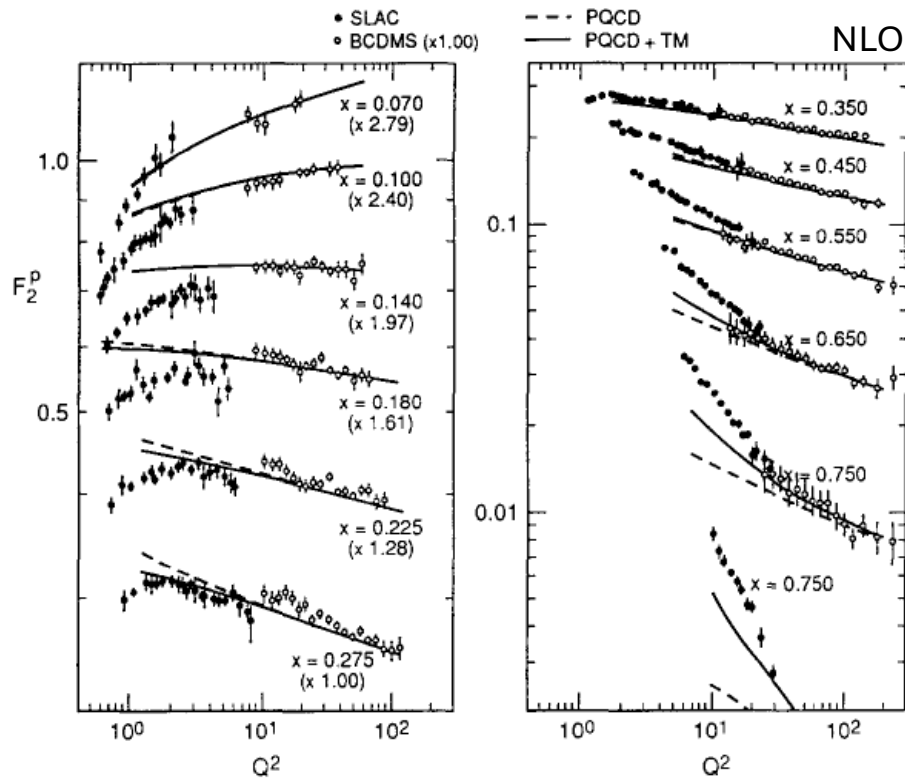
Received 17 October 1991

This comprehensive SLAC data can serve as precise anchors for the higher energy muon experiments.

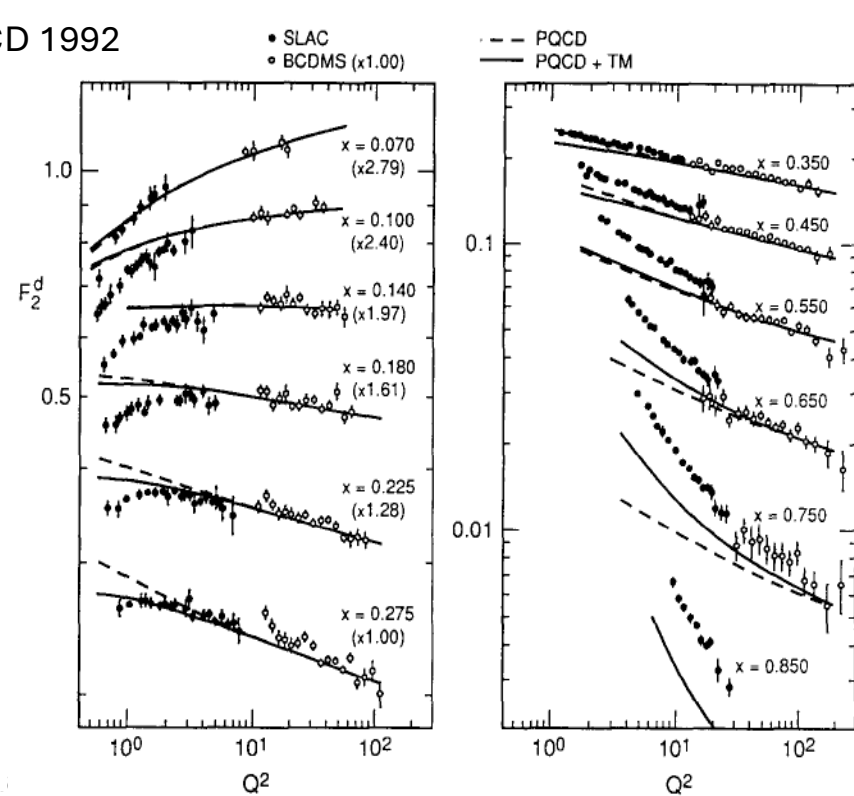
High energy muon experiments did not agree with SLAC. *Phys.Lett.B* 282 (1992) 475-482

Comparison to CERN BCDMS muon experiment. - Not Good. Both normalization and shape are not consistent,

BCDMS proton

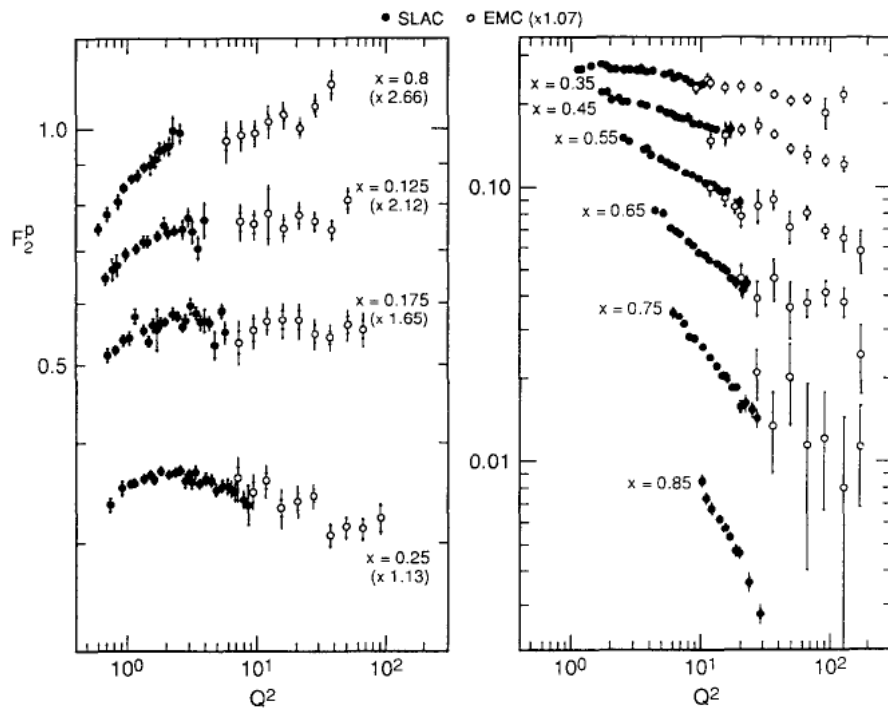


BCDMS deuteron

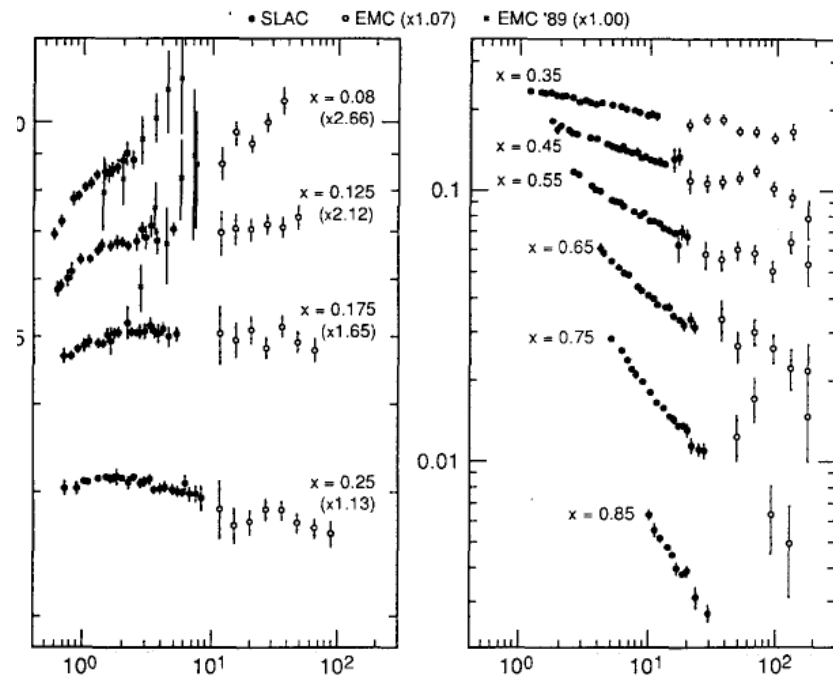


Comparison to CERN EMC muon experiment. – Not Good.

EMC proton (times 1.07)

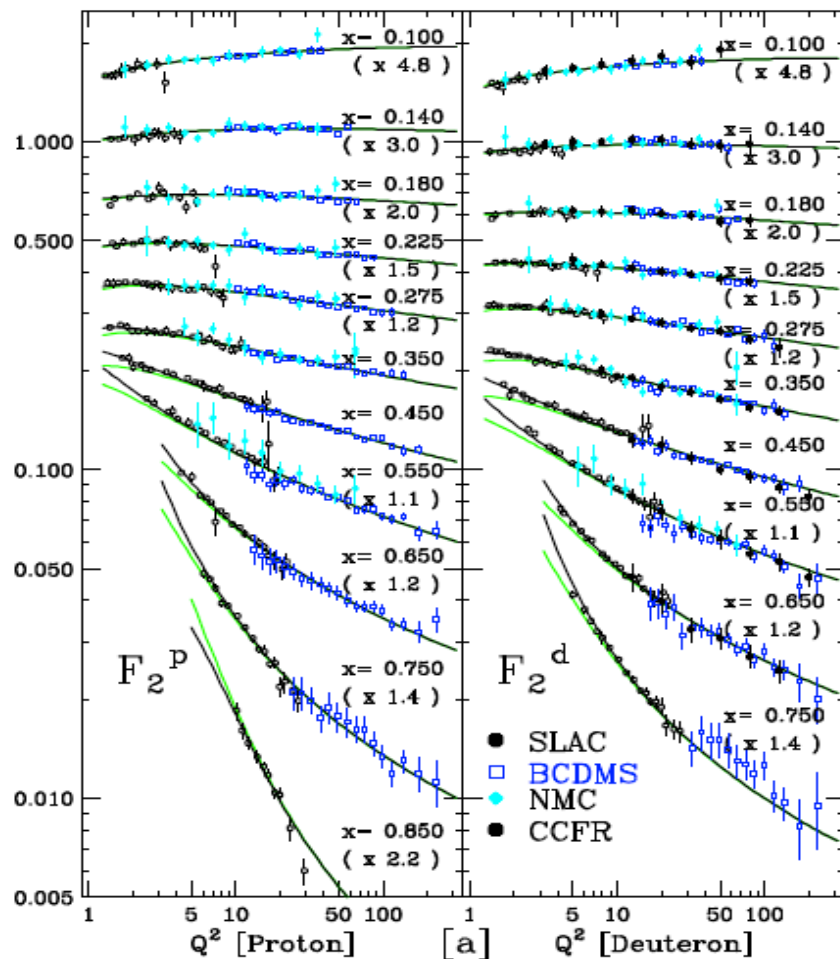


EMC deuteron (times 1.07)



NNLO QCD 2000

— NNLO (PQCD+TM+HT)
— NNLO (PQCD+TM)



Historical Example-continued

1992

- **BCDMS shape and normalization did not agree with SLAC**
- **EMC shape was better, but normalization was 7% too low.**

A unified analysis of all SLAC electron scattering experiments with **2% normalization uncertainty served as anchors for the higher energy muon scattering experiments**

The muon experiments took a long time to investigate and found unaccounted for systematic errors.

- BCDMS revised their magnetic field → leading to a change in Q^2 dependence and normalization
- EMC redid the reanalysis and revised their normalization.
- **New Muon Collaboration, NMC** measurement came later – they had measurements which overlapped SLAC and agreed with SLAC.

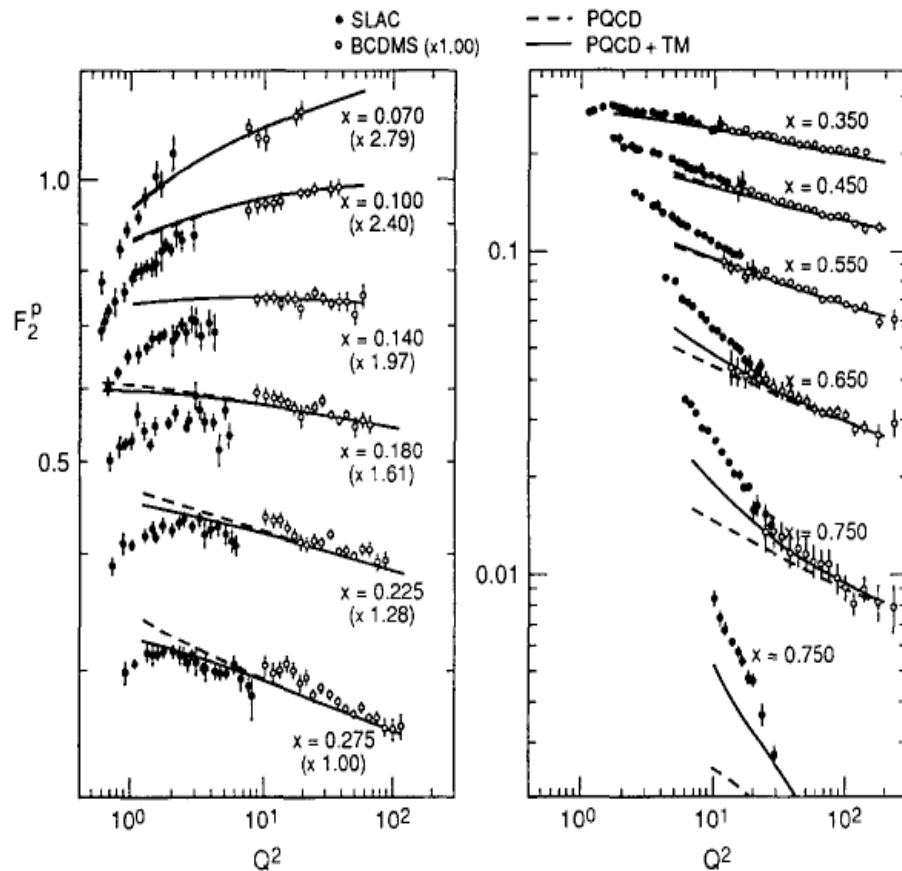
With these corrections, the low Q^2 electron scattering SLAC and the high Q^2 CERN EMC, BCDMS and NMC muon measurements became continuous (**NLO QCD 1999**) Arie Bodek, UK Yang, Phys.Rev.Lett.82:2467-2470,1999

NNLO PQCD+Target Mass describes all SLAC low Q^2 and CERN high Q^2 experiments, **Higher Twist contributions are small**. **NNLO QCD 2000** UK Yang, Arie Bodek, Eur.Phys.J.C13:241-245,2000

QCD fits before SLAC anchors were available

NLO QCD 1992.

BCDMS proton



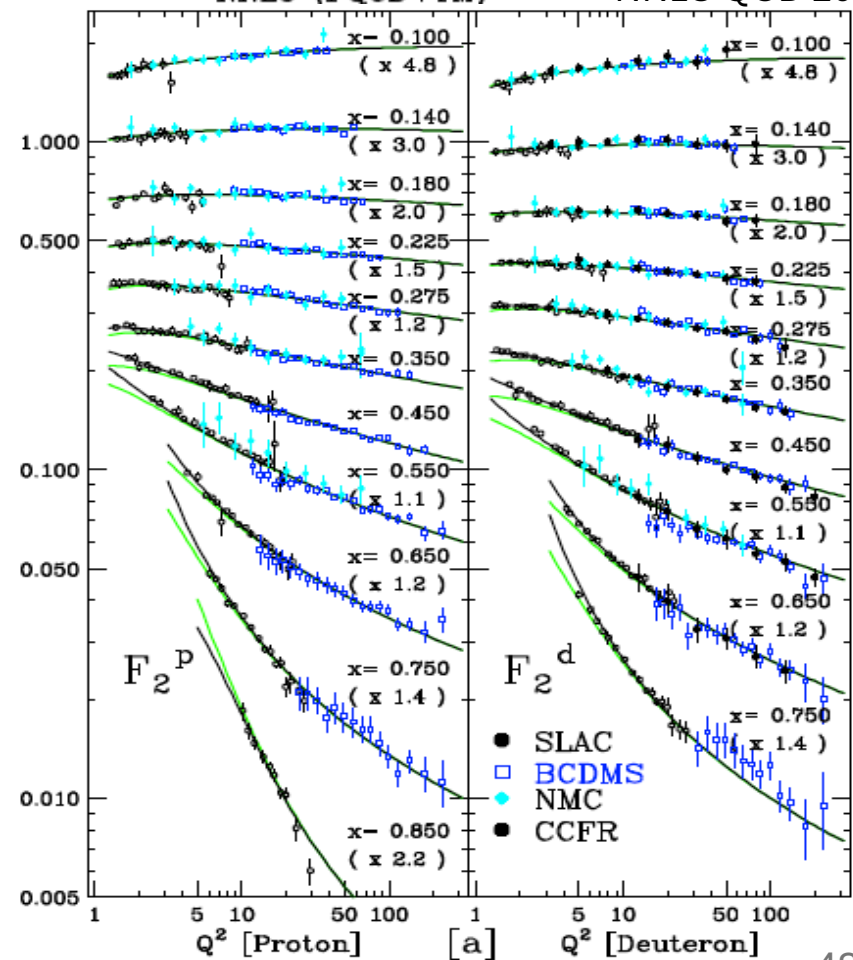
QCD fits After SLAC anchors became available

— NNLO (PQCD+TM+HT)

Tolerance = 1

— NNLO (PQCD+TM)

NNLO QCD 2000



What Anchors can be used for Global NNLO PDF fits

- SLAC F2 structure function data, low Q
- NMC (new muon collaboration) F2 structure functions at low Q
NMC agrees with SLAC in overlap region
- CMS. W asym 13 TeV. $Q=M_W$
- CMS W/Z . 13 TeV and 5 TeV. $Q=M_Z$
- CMS A4 Above Z mass (13 TeV). $Q=M_Z$
- ATLAS A4 from triple differential cross sections above Z mass (8 TeV). ($Q=M_Z$)

USING CMS 13 TeV PROFILED PDFs to REDUCE PDF ERROR IN LHCb and ATLAS

See what happens with CT18Z

profiled PDFs can be used to reduce the contribution of PDF uncertainty c to the uncertainty in mixing angle extracted from LHCb 13 TeV , and ATLAS 8 TeV Drell Yan Afb Data to 0.00015

PDF uncertainty in units of 10^{-5}

CMS13 TeV: **53 (no profiling)** -> **17 (CMS A4+Wasym+Rwz)**

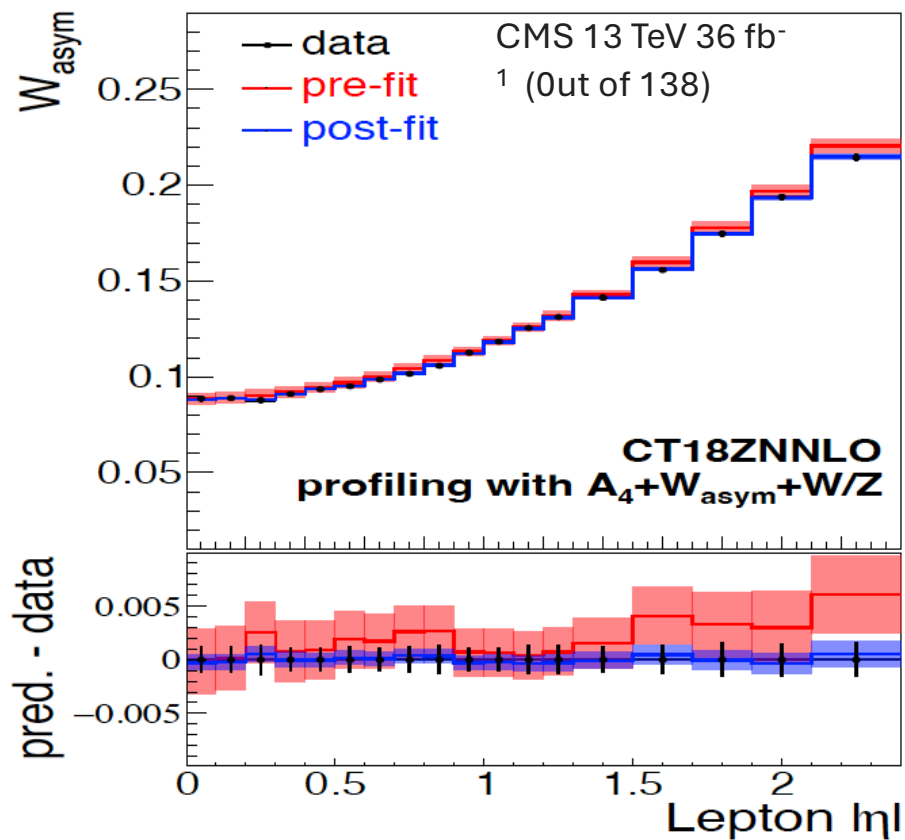
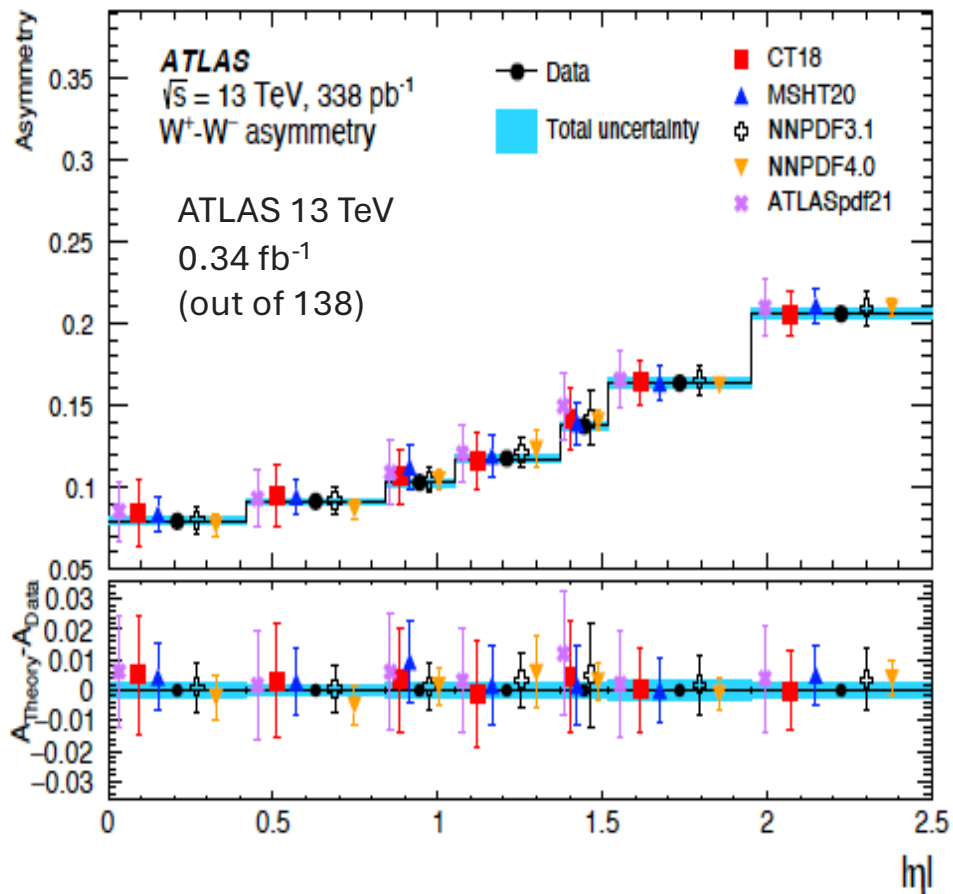
LHCb: **26 (no profiling)** -> 15 (CMS A4+Wasym+Rwz) -> **15 (CMS A4+Wasym+Rwz + LHCb $A_{FB}(\eta)$)**

ATLAS: **43 (no profiling)** -> 18 (CMS A4+Wasym+Rwz) -> **15 (CMS A4+Wasym+Rwz + ATLAS A_{FB})**

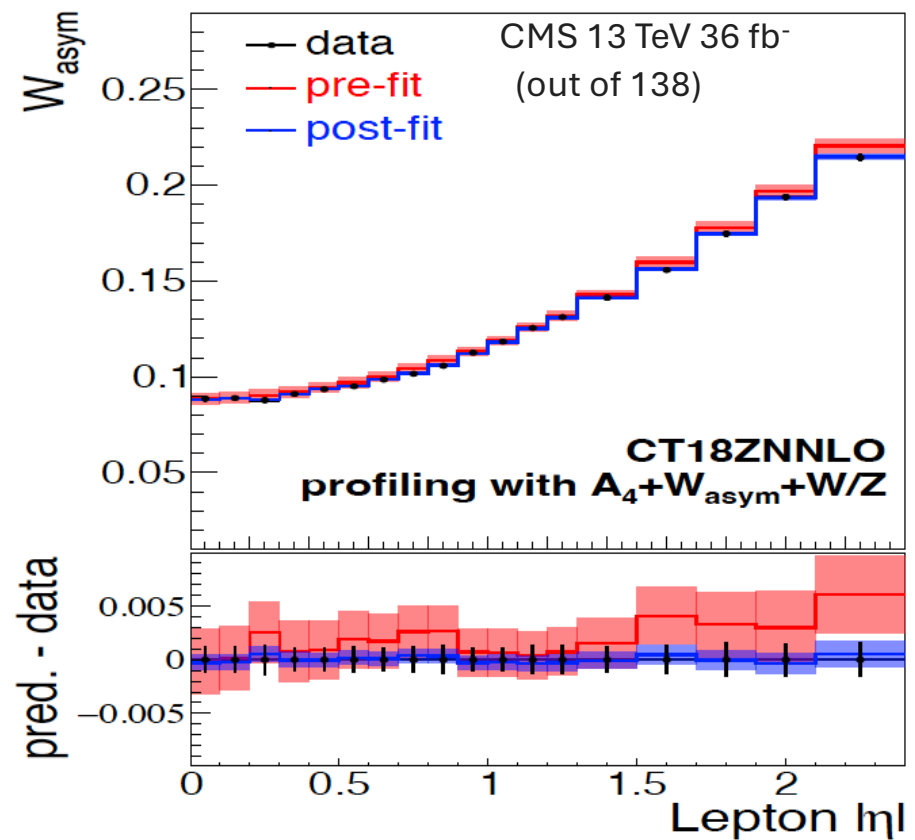
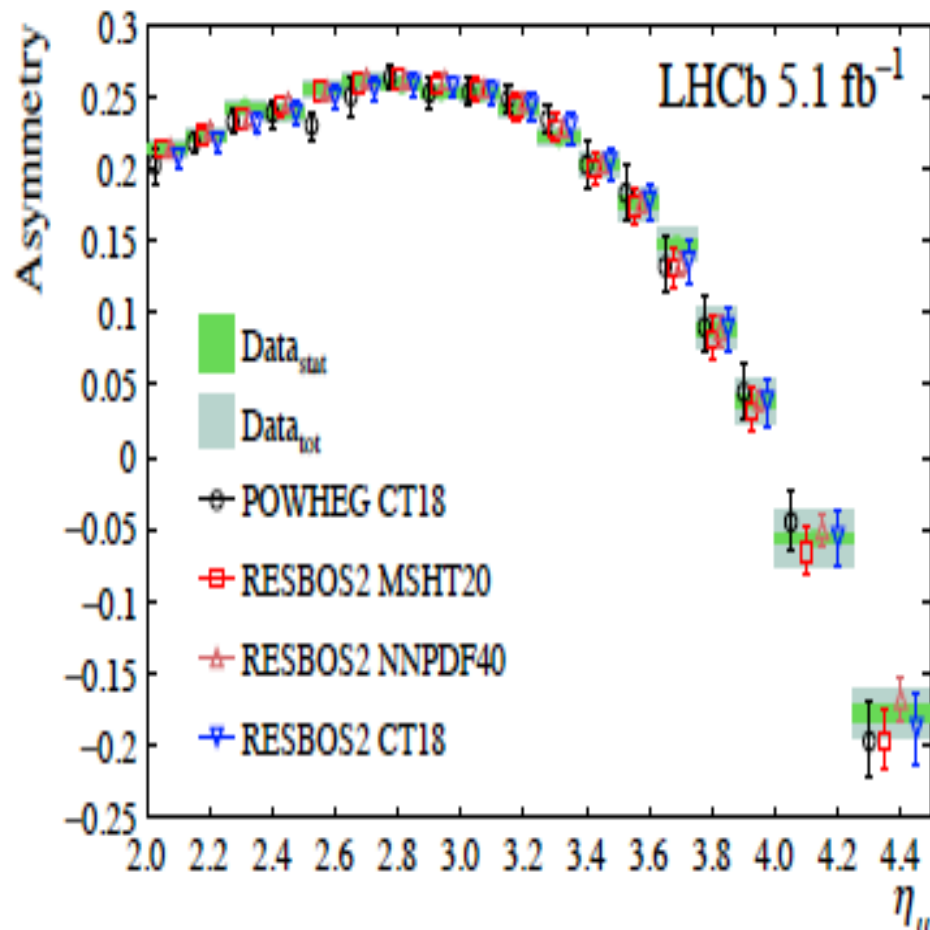
Here, ATLAS Drell Yan A_{FB} above and below Z mass helps.

We are working on LHC combination of the above three: expect total error about **21×10^{-5}**

ATLAS 13 TeV measurements of the W asymmetry is based on 0.34 fb⁻¹ of data only, So it has no constraining power.



LHCb. W asymmetry is also not as precise as CMS at 13 TeV (so it has constraining power)



LHCb

$$0.23147 \pm 0.00044(stat) \pm 0.00005(exp) \pm 0.00021(PDF) \pm 0.00009(th)$$

CT18ZNNLO

	total unc.	PDF unc.
NoProfiling	0.00051	0.00026
Profiling	0.00051	0.00026
NoProfiling+iCMSPDF	0.00046	0.00015
Profiling+iCMSPDF	0.00046	0.00015

NNPDF31_nnlo

	total unc.	PDF unc.
NoProfiling	0.00049	0.00023
Profiling	0.00049	0.00023
NoProfiling+iCMSPDF	0.00047	0.00018
Profiling+iCMSPDF	0.00047	0.00018

PDF set	$\sin^2 \theta_{\text{eff}}^{\ell}$	PDF uncertainty	Shift	Fit χ^2/ndof
NNPDF31_nnlo.as0118	0.23155	0.00023	–	8.4/9
CT18NLO	0.23165	0.00022	+0.00009	8.4/9
MSHT20nnlo.as118	0.23137	0.00017	–0.00018	8.2/9
Arithmetic average	–	0.00021	–0.00003	–

- NoProfiling: no PDF constraint
- Profiling: PDF constraint from LHCb AFB
- NoProfiling+iCMSPDF:
 - PDF constraint from CMS A4+Wasym+Rwz
- Profiling+iCMSPDF
 - PDF constraint from LHCb AFB + CMS A4+Wasym+Rwz
- iCMSPDF = profiled PDF with CMS A4+Wasym+Rwz
- PDF uncertainty reduction depends on PDF set
 - CT18Z: 26- > 15
 - NNPDF31: 23- > 18
- **LHCb used NLO PDFs**
 - average uncertainty of CT18, MSHT20, NNPDF31: 0.00021
 - I don't have the numbers for NLO PDFs now

ATLAS

Our own analysis of ATLAS 8 TeV Tripple differential Drell-Yan Cross Sections

$$0.23140 \pm 0.00021(stat) \pm 0.00015(exp) \pm 0.00024(PDF) \pm 0.00007(th)$$

NNPDF3.1 Profilingwith
ATLAS A4 data only

CT18ZNNLO

	total unc.	PDF unc.
NoProfiling	0.00052	0.00043
Profiling	0.00040	0.00027
NoProfiling+iCMSPDF	0.00034	0.00018
Profiling+iCMSPDF	0.00033	0.00015

- CT18Z: 27->15
- NNPDF31: 24->18
- **ATLAS nominal PDF was MMHT14**
 - I don't have the numbers for MMHT14
 - Uncertainty for NNPDF31 agrees well

NNPDF31_nnlo

	total unc.	PDF unc.
NoProfiling	0.00051	0.00042
Profiling	0.00038	0.00024
NoProfiling+iCMSPDF	0.00036	0.00020
Profiling+iCMSPDF	0.00034	0.00018

	CT10	CT14	MMHT14	NNPDF31
$\sin^2 \theta_{\text{eff}}^{\ell}$	0.23118	0.23141	0.23140	0.23146
Uncertainties in measurements				
Total	39	37	36	38
Stat.	21	21	21	21
Syst.	32	31	29	31

CMS only 13+13.6 TeV combination

CMS Run2. 13 TeV. 140 fb⁻¹. Published

	$\chi^2_{\min}$	bins	p (%)	$\sin^2 \theta_{\text{eff}}^{\ell}$	stat	exp	th	PDF	Exp= 0.00015				
									MC	bg	eff	calib	other
$\ell\ell$	731	816	98	$23\,152 \pm 31$	10	15	8	27	8	4	6	6	3

MC statistics is large Exp systematic error.

$$\sin^2 \theta_{\text{eff}}^{\ell} = 0.23156 \pm 0.00024. \text{ (0.00010 stat., 0.00015 syst., 0.00008 theory, 0.00017 PDFs)}$$

(with the additional PDF constraints from W_{asym} and W/Z cross section ratio)

run2 A4: 31 $\rightarrow$ Run2 A4+Wasym+Rwz: **24**

If we scale both data and MC stat by $\sqrt{140 \text{ fb}^{-1}/440 \text{ fb}^{-1}}$,

run2+3 A4: 29. $\rightarrow$ run2+3 A4+Wasym+Rwz: **21**

If we scale data stat by $\sqrt{140/440}$ and assume zero MC stat error (Large MC limit),

run2+3 A4: 28. $\rightarrow$ Run2+3 A4+Wasym+Rwz: **20**

$\sin^2 \theta_{MS}^l (\mu)$ at high mass.

- The Rochester-SNY group is also working on the measurement of $\sin^2 \theta_{MS}^l (\mu)$ above the Z mass (up to 3 TeV) with CMS combined Run2 13 TeV and Run3 13.6 TeV Drell-Yan data.
 - Afb (M,y) is **not very sensitive to $\sin^2 \theta_{MS}^l (\mu)$** $\rightarrow$ But Afb at high mass provides additional PDF constraints
 - $[d\sigma/dMdy]$ is sensitive to both PDFs and **$\sin^2 \theta_{MS}^l (\mu)$** .
 - In order to reduce experimental (luminosity, acceptance efficiency()) and theoretical uncertainties (higher order) **propose using**
NormXSEC= $[1/\sigma_Z] [d\sigma/dMdy]$
 - **Fit both: mass dependent Afb and mass dependent NormXSEC**

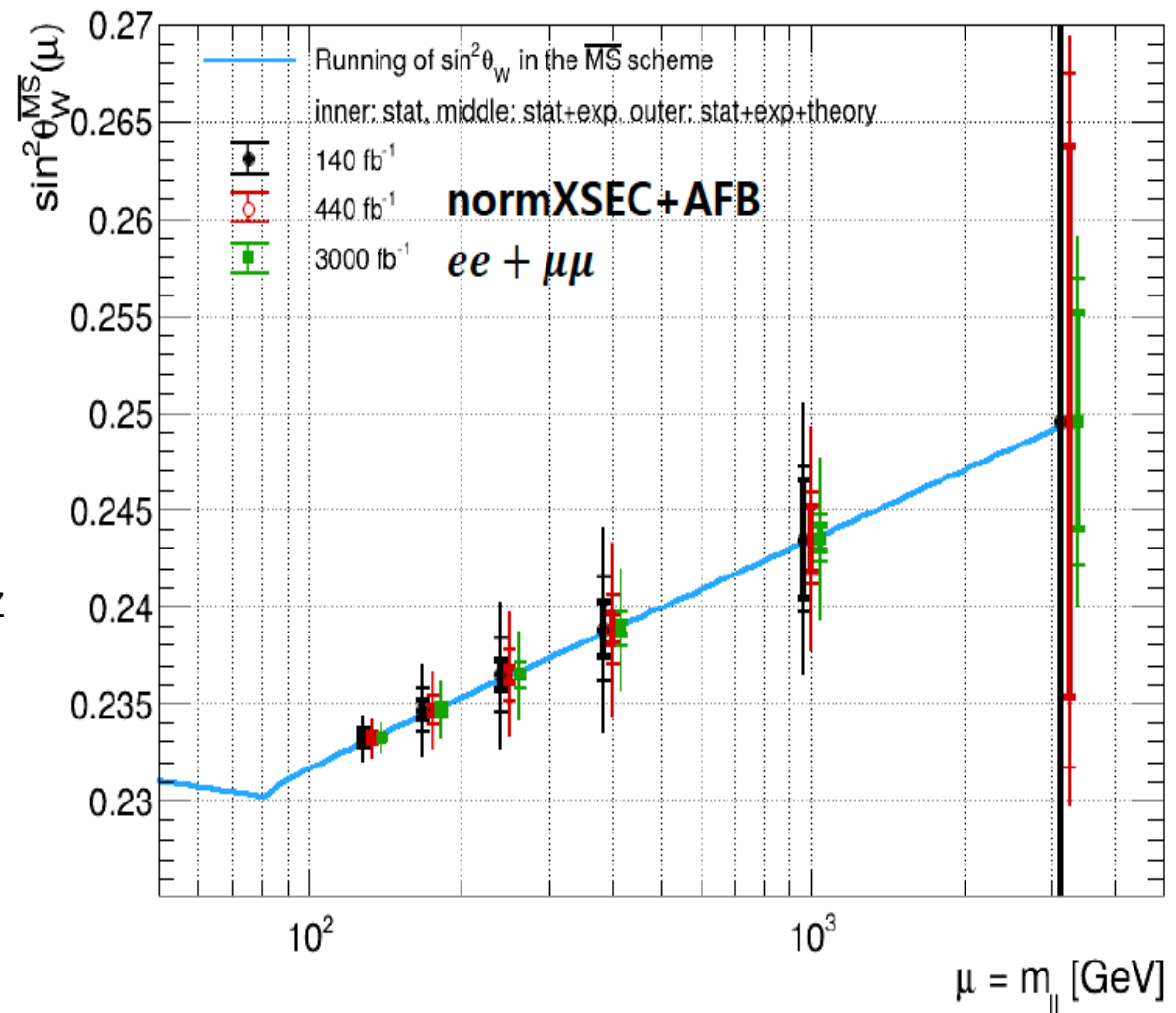
1. Expected sensitivity with Run2 13 TeV data. 140 fb⁻¹
2. Run2+Run3 13.6 TeV (440 fb⁻¹)
3. High Luminosity Run in the future. 3000 fb⁻¹

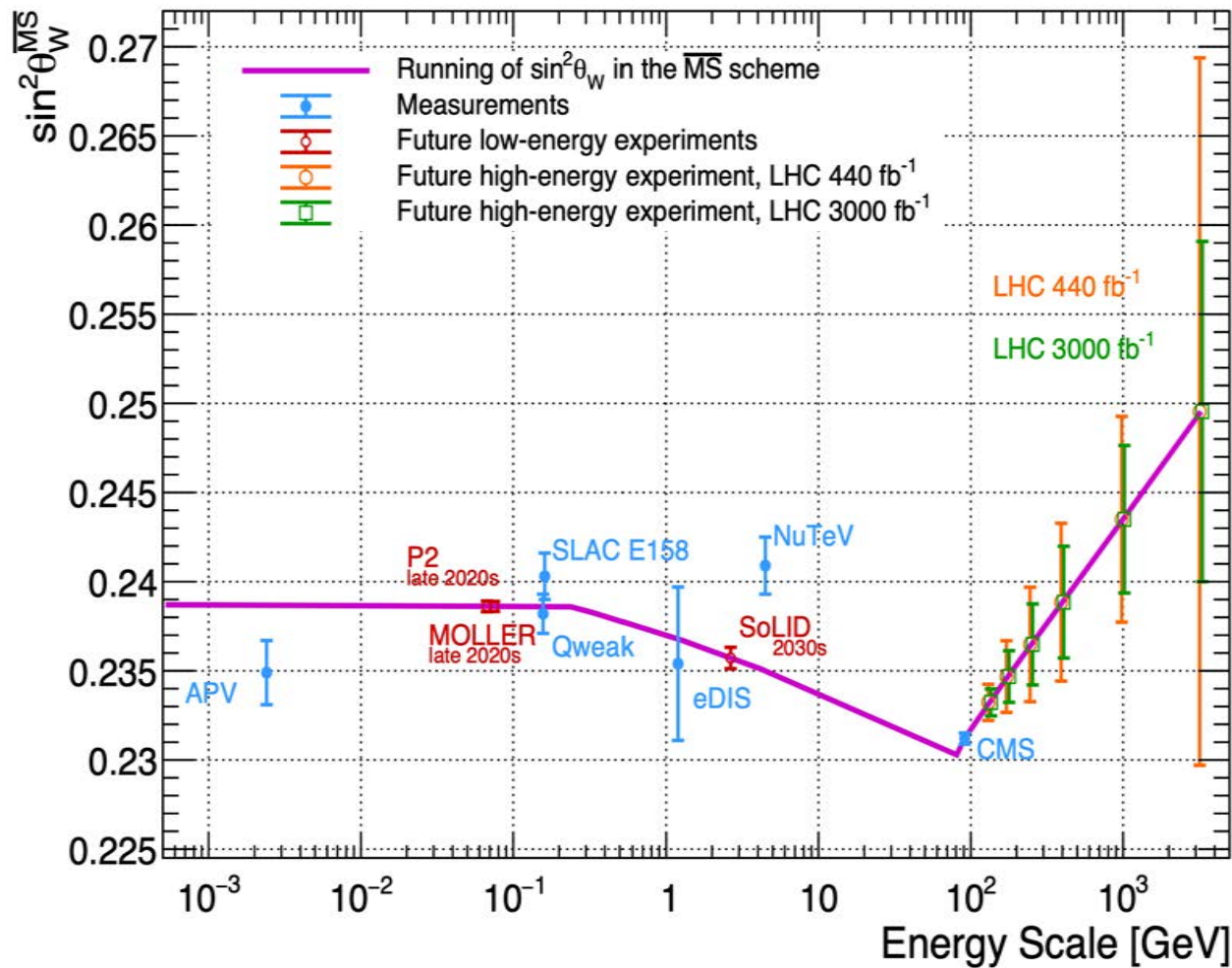
Run2+Run3 is 20% better than Run2 only.

Use

1. Drell Yan cross $d\sigma/dM dy$ normalized to the Z cross sections. (most sensitive to mixing angle)
2. Afb (M,y)

Here also , use ratios and asymmetries.





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Measurements at different energy scales

$$\sin^2 \theta_W^{\overline{\text{MS}}}(\mu)$$

- Low-energy measurements
 - Several measurements with limited precision (~ 0.001)
 - Some future experiments with high precision (~ 0.0003)
- Z-pole measurements
 - LEP, SLD, Tevatron, LHC
 - CMS SMP-22-010
 - ~ 0.0003
- High-energy measurements
 - No measurement yet

With current data

- LHC 440 fb⁻¹ (13+13.6 TeV) est.

With future data

- LHC 3000 fb⁻¹ est.

APV, Qweak, E158, eDIS, NuTeV: [PhysRevD.110.030001](https://arxiv.org/abs/1103.03001)

P2: [arXiv:1802.04759](https://arxiv.org/abs/1802.04759)

SoLID: [arXiv:1302.6263](https://arxiv.org/abs/1302.6263)

MOLLER: [arXiv:1411.4088](https://arxiv.org/abs/1411.4088)

CMS: [arXiv:2408.07622](https://arxiv.org/abs/2408.07622)

LHC high mass: [arXiv:2302.10782](https://arxiv.org/abs/2302.10782)

Conclusion

- We have entered a new era of precision EW measurements at LHC.
- W mass and weak mixing angles are in very good agreement with SM.
- Continue search for deviation from SM in measurements of weak mixing angle at high mass (up to 3 TeV).

Backup

Unfolding for detector efficiency, resolution ,acceptance and FSR

Extracting unfolded A4 (M,y)

- **Needed for a reliable combined analysis of CMS, ATLAS, LHCb.**
- **Can be used in future extraction of the mixing angle with better PDFs, better PDF profiling , and analysis with more sophisticated higher order theory models, by any experimentalist or theorist.**
(as was done in 2025 updated analysis of CMS data)
- **In CMS this was done by fitting the observed $\cos\theta$ distribution of events: yield based fit. CMS published unfolded A4 at 13 TeV. ATLAS published unfolded triple differential cross sections at 8 TeV.**
- This method of unfolding requires *precise determination of lepton detection efficiencies, resolution acceptance, luminosity.*

Detector level measurement as a check on unfolding

If the **angular-weighted Afb** is used, the systematic **uncertainties mostly cancel**, so an initial efficiency determinations suffice. To extract **$\sin^2 \theta_{eff}^l$** requires detector level theory predictions, and the analysis can only be done by the experimenters. This can be done relatively quickly,

- In CMS we had to improve determination of efficiencies and **resolutions until the extracted $\sin^2 \theta_{eff}^l$ from the unfolded A4 agreed with detector level angular-weighted Afb $\sin^2 \theta_{eff}^l$ measurement**
- **It took about 1.5 additional years to improve modeling efficiencies and resolutions** for muon, central electrons, forward electrons with tracking, forward electrons with no tracking, and electrons in the forward HF.
- Having the detector level angular-weighted Afb **$\sin^2 \theta_{eff}^l$** measurement was essential.

Summary of A4 unfolding at CMS with $\cos \theta_{CS}$ yield-based fit -

- CMS extracted $A_4(m, |y|)$ using a $\cos \theta_{CS}$ yield-based **template fit**
$$\text{Yield}(\text{Period}, \text{Channel}, m, |y|, \cos \theta_{CS}) \rightarrow A_4(m, |y|)$$
 - **Input:** event yields in (Period, Channel, m , $|y|$, $\cos \theta_{CS}$) bins, total 14205 bins
 - **Output:** 63 pre-FSR $A_4(m, |y|)$
- Main challenge: The yield-based fit is challenging because good detector and theory modeling are required to avoid degraded fit quality or biased A_4 extraction
- Practical solution in CMS 13 TeV
 - Substantial effort in improving detector and background modeling. (1.5 years)
 - 38 floating differential-yield scale factors $\kappa(m, |y|)$ to absorb residual differential-yield mismodeling
- Questions
 - The need for $\kappa(m, |y|)$ suggests residual mismodeling in the differential yields, possibly mismodeling of the DY $\sigma(m, |y|)$ prediction – Need to compare to higher order cross section theory predictions (to be investigated)
 - The $\kappa(m, |y|)$ parameters are common across channels and data-taking periods. The fact that we still obtain reasonable consistency among channels and periods suggests that the observed yield mismodeling is not mainly driven by detector- or period-specific effects, but is more likely related to the modeling of the differential DY cross section. -

Alternative: Unfolding by fitting the $\cos\theta$ dependence of Afb only

- A4 unfolding can also be done with angular Afb-based template fit, which we plan to use at CMS at 13.6 TeV

$$A_{FB}(\text{Period, Channel}, m, |y|, \cos \theta_{CS}) \rightarrow A_4(m, |y|)$$

- Fit is not sensitive to absolute reconstructed event-yield
 - Stable against differential yield mismodeling (absolute yields cancels)
 - Experimental systematic errors also cancel in angular Afb - so perfect modeling of efficiencies not needed
 - Then, extract the $\sin^2 \theta_{\text{eff}}^l$ from unfolded A4 using higher order models for which Monte Carlo generators are not available.
 - It can be done relatively quickly

=====

- To investigate differential-yield mismodeling – one can perform a yield based A4 unfolding as a second step
- Requires work on efficiency modeling to make sure that one gets the same A4 as the angular Afb based unfolding.

Profiled PDF and Tolerance – mixing angle analysis

- My opinion is that tolerance factors for Precise Ratio and Afb measurements should be 1.0 in all global PDF analyses since the errors are primarily statistics.
- These could serve as precise candles to discover un-accounted for systematic errors in other experiments (as the SLAC electron scattering data was to discover systematic errors in the CERN muon experiments)
- Precise measurements also test the flexibility of the parameterizations.

There are cases in which experimenters want to constrain PDFs such that the predictions agree for all distribution measured by the same detector for a particular experimental analysis in a particular kinematic region

The argument: Can an experiment use PDFs which do not describe their data?

Some groups made the choice extract their own PDFs instead (was done by ATLAS, HERA). I think a better choice is to profile a large number of PDF sets and see how much they differ from each other,

For the W mass and electroweak mixing angle, the most important consideration is to use the most precise PDFs in the W/Z kinematic region, which is why we are using a tolerance factor of 1.0 for the W and Z Boson data.