SECRET STUDENT NUMBER: ____________________

Classical Mechanics Subject Exam   August 28, 2017

DO NOT WRITE YOUR NAME ON ANY SHEET!

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1. [10 pts] The position of a point particle is governed by the equation of motion

$$\ddot{x} + 4x = \cos(\omega t)$$

(a) Solve this differential equation to find x as a function of time t, given that the particle starts from rest at the point $x = 0$ at $t = 0$.

(b) Take the limit $\omega \to 2$ in your answer to part (a) and describe the solution in that limit.

2. [10 pts] A particle of mass M slides without friction inside the surface of a frictionless paraboloid of revolution $z = A(x^2 + y^2)$ where $A > 0$ is a constant. The symmetry axis z of the paraboloid is vertical, so there is a gravitational force in the $-\hat{z}$ direction.

   (a) Find the Lagrangian, using the polar coordinates r and ϕ as generalized coordinates, where $x = r\cos\phi$ and $y = r\sin\phi$.

   (b) There are two obvious constants of the motion. Use those constants to find $\dot{r}$ as a function of r.

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3. [10 pts] Find the path $y(x)$ from $(0,0)$ to $(1,1)$ in the (x,y) plane that makes the integral

$$I = \int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} + 4y^2 \right] dx$$

a minimum. This is a calculus of variations problem: you should write the Euler-Lagrange equation, solve it, and then impose the end-point conditions.

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4. [10 pts] Consider the Lagrangian $L = \frac{1}{2}M(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + K\,x\,\dot{y}$ where x and y are generalized coordinates and K and M are constants.

   Change variables in this Lagrangian to cylindrical coordinates $x = r\cos\phi$, $y = r\sin\phi$, $z = z$. Then find the Hamiltonian as a function of the new coordinates r, ϕ, z and their appropriate canonical momenta.