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QUANTUM MECHANICS

Subject Exam

Total $4 \times 20 + 5 \times 4 = 100$ points

May 1, 2017

PROBLEM 1. Establish the uncertainty relation for the Heisenberg position operators $\hat{x}(t)$ and $\hat{x}(t')$ taken at different times for a particle of mass m in the one-dimensional harmonic oscillator potential of frequency ω.

PROBLEM 2. A pulse of the electric field,

$$\mathcal{E}_x(t) = \mathcal{E}_0 e^{-(t/\tau)^2},$$

is applied to a particle of mass m originally in the ground state of the one-dimensional harmonic potential of frequency ω. Using the perturbation theory, find the probability for the particle to be in an excited state after the pulse. Compare the efficiency of energy transfer for two pulses supplying the same total momentum but differing in the value of the parameter τ if both pulses justify the use of perturbation theory.

PROBLEM 3. A particle of mass m is placed in a one-dimensional potential well of finite depth U_0. The width a of the well is fixed in such a way that the particle has only one bound state with binding energy at the middle depth, $\epsilon = U_0/2$. Calculate the probabilities of finding the particle in classically allowed and classically forbidden regions.

PROBLEM 4. For the free motion of a relativistic electron, define the operator of velocity, find the eigenvalues of this operator and its expectation value for the actual function of free motion with certain energy.

Additional questions.

1. Argue if the following transitions are allowed in the dipole approximation:

   a. in the helium atom between a state of orthohelium (spin of two electrons is $S = 1$) and a state of parahelium (spin $S = 0$);

   b. between the components of fine atomic structure derived from the same LS-term;

   c. between the components of hydrogen hyperfine structure.

   If a transition is forbidden, suggest what type of the radiation is possible in that case.

2. A quantum level of a system with the total angular momentum J is split by the weak static uniform field along a certain axis.

   a. Explain the difference (if any) in the character of splitting between the cases of electric and magnetic field.

   b. Describe the splitting of the atomic term ${}^6G_{3/2}$ in the weak magnetic field.

3. The nucleus of the oxygen isotope ^{18}O has two valence neutrons on the $d_{5/2}$ orbital. In the approximation of the inert core of ^{16}O, find the total number of quantum states allowed in this configuration and allowed values of the total angular momentum and parity.

4. Many researchers in various laboratories study the Bose-Einstein condensation of identical neutral atoms in atomic traps. This phenomenon starts when the average distance between the atoms is comparable to their de Broglie wavelength of thermal motion.

   a. Estimate the necessary density of atoms with the mass number $A = 100$ in the trap at a temperature $T = 10^{-7}$K.

   b. Which of the rubidium isotopes (nuclear charge $Z = 37$),^{87}Rb or ^{86}Rb, should be taken for such experiments?,

5. The electrons emitted from an atomic process are described by the wave function

$$\psi(\mathbf{r}) = \frac{e^{ikr}}{r} (\hat{\mathbf{s}} \cdot \mathbf{n}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{1}$$

where $\hat{\mathbf{s}}$ is the spin operator, $\mathbf{n} = \mathbf{r}/r$ the unit vector, and the column is the spinor in the z-representation. Determine the quantum numbers of this state (orbital momentum ℓ, total angular momentum j, its projection j_z, parity) and the angular distribution of emitted electrons.