Number ....

QUANTUM MECHANICS

Subject Exam

Total $4 \times 20 + 5 \times 4 = 100$ points

April 30, 2018

PROBLEM 1. *a.* A quantum harmonic oscillator with mass m and elastic force $-kx$ is perturbed by a small change $k \to k + \xi$. Using perturbation theory find the change of the energy spectrum in the lowest non-vanishing order.

b. Find the exact solution for the perturbed oscillator and compare the result to that of perturbation theory.

PROBLEM 2. Two identical atoms of zero spin are placed in the spherical harmonic oscillator trap. At given temperature, only the shells $n = 1, 2, 3$ in the trap are accessible for the atoms. List all posible configurations of the system and the allowed values of the total angular momentum L of the system in each configuration. Check your answer by counting the total number of possible states and comparing to the total number expected by the statistics. Count how many of these states have negative parity.

PROBLEM 3. An electron moving along the z-axis with energy E is scattered by the pair of point-like charged centers located at $z = 0$ and $z = a$ (along the same axis); the centers have the charges e'. Using the Born approximation find the elastic scattering cross section as a function of the scattering angle θ. Determine the scattering angles corresponding to the maxima of the cross section.

PROBLEM 4. *a.* Find the commutation relation between different components $\hat{v}_i$ of the velocity operator for the Dirac particle. Recall the similar commutator for transverse components of velocity for a non-relativistic particle of mass m and electric charge e in the static uniform magnetic field. By comparison of the commutation relations, find the effective magnetic field created by the spin of the moving Dirac particle.

b. Estimate the interaction energy of the magnetic moment equal to the Bohr magneton for this particle with this effective magnetic field.

PROBLEM 5a. An electron is confined in the lowest s-state inside an empty spherical shell of radius R. Determine the pressure exerted on the surface of the sphere.

PROBLEM 5b. For the lowest electron state in the hydrogen atom, estimate magnitude of the electric field of the nucleus acting on the electron at this orbit (in V/cm).

PROBLEM 5c. Fine-structure of energy levels in a complex atom is described by the Hamiltonian

$$\hat{H}' = A(\hat{\mathbf{L}} \cdot \hat{\mathbf{S}}), \tag{1}$$

where $\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}$ are operators of the total orbital momentum and the total spin of electrons, respectively.

a. Draw the level scheme for $L = 3$ and $S = 2$ if $A < 0$; indicate spectroscopic symbols for the split levels.

b. Find the level J that does not split in a weak magnetic field.

PROBLEM 5d. Vector particles (spin $s = 1$, negative intrinsic parity, for example $\rho - meson$) decay into two pions, $\pi^+\pi^-$ (pions are spinless). The ρ-mesons were created in a previous reaction with their spin projection $s_z = +1$ along a certain axis. Find the angular distribution of the pions if the decay is driven by strong interactions.

PROBLEM 5e. A wave function of the particle depends on the azimuthal angle φ with respect to the polar axis z as

$$\psi(\varphi) = \mathrm{const} \cdot \sin^2 \varphi. \tag{2}$$

Determine the probabilities of various values of the orbital momentum projection $\ell_z = m$.