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Qualifying/Placement Exam, Part-A
10:00 – 12:00, August 17, 2017, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{eV\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\mathrm{C}$	charge of the electron

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1. [10 pts] The Poisson distribution gives the probability that an integer number of events, x, occur in unit time when the mean rate of occurrence is μ:

$$f_{\mathrm{P}}(x) = \frac{e^{-\mu}\mu^{x}}{x!}.$$

a) [4 pts] Show that $f_{\mathrm{P}}(x)$ is normalized; include a derivation of any series expansion required.

b) [2 pts] Show that the mean of $f_{\mathrm{P}}(x)$ is μ.

c) [2 pts] Show that the variance of $f_{\mathrm{P}}(x)$ is μ.

d) [2 pts] To date, no proton decay events have been observed. Assume that the rate of proton decay in a large sample of protons is 3 per year, what is the probability that we will see no decays in one year?

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2. [10 pts] Consider the matrix,

$$M = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}.$$

a) [5 pts] What are the eigenvalues of M ?
b) [5 pts] Find the inverse matrix M^{-1}.

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3. [10 pts] Consider the function,

$$f(x) = \left.\frac{a}{a^2 + x^2}\right|_{a \to 0}.$$

a) [5 pts] Prove this function is proportional to the delta function, $\delta(x)$.

(Hint: you may find useful the substitution, $x = a\tan\theta$)

b) [5 pts] Determine the constant proportionality *C*, i.e.,

$$\left.\frac{Ca}{a^2 + x^2}\right|_{a \to 0} = \delta(x).$$

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4. [10 pts] Consider a (conducting) solid metal sphere.

a) [5 pts] If the volume of the metal sphere is tripled, by what factor does its capacitance change?
b) Assume the sphere carries a charge, Q. At the sphere's final radius, R, determine how much energy is stored in two ways:

- i) [3 pts] using the potential and capacitance of the sphere, and
- ii) [2 pts] using the electric field outside the sphere.

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5. [10 pts] Consider a cylindrical conducting shell of radius a, and infinite length. It is oriented with its axis along the z-axis in a region that originally had a constant electric field $\vec{E} = E_0\hat{x}$.

a) [5 pts] Find the electric potential outside the shell, i.e., for radius $\rho = \sqrt{x^2 + y^2} > a$.
b) [3 pts] Find the electric potential inside the shell.
c) [2 pts] Find the surface charge density on the shell.

Helpful Information: The general solutions to Laplace's equation (with no z dependence, $k_z = 0$) in cylindrical coordinates are

$$\Phi(\vec{\rho}) = C + D\ln(\rho) + \sum_{m\neq 0} e^{im\phi}\left\{\frac{A_m}{\rho^m} + B_m\rho^m\right\}, \quad \rho = \sqrt{x^2 + y^2}$$

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6. [10 pts] A small circular current loop of radius R carrying a current I is placed in a region of constant magnetic field B. Originally, the axis of the loop is lined up with the direction of $\vec{B}$ to minimize the energy. The loop is then rotated so that the axis makes an angle θ relative to the $\vec{B}$ direction. Calculate the magnitude of the torque acting on the loop. Best to begin with $d\vec{F} = I\, d\vec{l} \times \vec{B}$, and define your coordinate system.