Student No.: ______________________

Qualifying/Placement Exam, Part-B
2:00 – 4:00, August 20, 2018, 1400 BPS

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99 \times 10^9\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4 \times 10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05 \times 10^{-34}\ \mathrm{J \cdot s}$	Planck's constant
$= 6.58 \times 10^{-16}\ \mathrm{eV \cdot s}$	"
$c = 3.0 \times 10^8\ \mathrm{m/s}$	speed of light
$e = 1.602 \times 10^{-19}\mathrm{C}$	charge of the electron
$N_A = 6.022 \times 10^{23}\ \mathrm{mol^{-1}}$	Avogadro constant

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1. [10 pts] Hydrogen wave functions are written as, ψ_{nlm}, where *n*, is the principle quantum number, *l*, the angular momentum quantum number, and *m*, the angular momentum projection quantum number. Consider the wave function, $\psi(r,0)$ of an electron in a hydrogen atom at $t = 0$:

$$\psi(r,0) = \frac{1}{5}\left(3\psi_{100} + \psi_{210} + 2\sqrt{2}\psi_{211} + 2\psi_{21-1} + \sqrt{3}\psi_{310}\right) .$$

The electron ground state energy in a hydrogen atom is E_1 = -13.6eV, and in a non-relativistic model the energy of any eigenstate, E_n, is a function of just E_1, and n.

a) [3 pts] In an eigenstate of the hydrogen atom, what function relates an electron's energy to its ground state energy?

b) [2 pts] Calculate the energy expectation value (in eV) of the electron in the state $\psi(r,0)$.

c) [2 pts] What is the time dependence of the electron's wavefunction, $\psi(r,t)$?

d) [3pts] Calculate the probability of finding the electron with the quantum number, $l = 1$.

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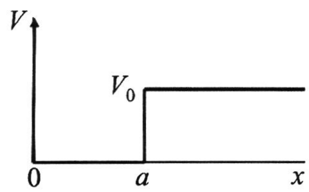
2. [10 pts] Consider a particle of mass m, bound in the potential as shown in the figure, where



$$V = \begin{cases} \infty, x < 0 \\ 0, 0 \le x \le a \\ V_0, a < x \end{cases}$$

a) [4 pts] For a bound state energy ($E < V_0$), display the Schrodinger Eq. in theregions of finite potential and state the boundary conditions on the bound state wavefunction, $\psi(x)$, at $x = 0$, and $x = +\infty$.

c) [4pts] Solve for $\psi(x)$ in these regions, and on the figure above sketch it for all $x > 0$.

d) [2 pts] Using the boundary conditions at the point $x = a$, show that the bound state energy, E, can be expressed as the solution to the equation, $\tan(\frac{\sqrt{2mE}a}{h/2\pi}) = -\sqrt{\frac{E}{V_0 - E}}$.

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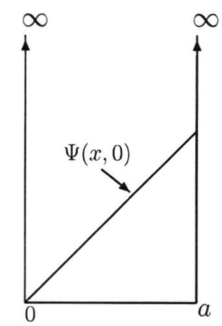
3. [10 pts] A particle of mass m moving in the infinite potential well shown in the figure has an initial (normalized) wave function $\Psi(x,0)$ given by



$$\Psi(x,0) = \begin{cases} Ax & \text{if } 0 \le x < a, \quad \text{where } A = \sqrt{3/a^3} \\ 0 & \text{otherwise} \end{cases},$$

a) [3 pts] Find $\langle x \rangle$.

b) [3 pts] What is the probability that the particle will be found in the interval $a/2 \le x \le a$?

c) [4 pts] If the energy of the particle is measured, what is the probability that the result will be

$$E = \pi^2\hbar^2/(2ma^2)?$$

Note: The normalized eigenfunctions of the infinite square well are $\psi_n(x) = \sqrt{\frac{2}{a}} \sin(n\pi x / a)$.