Student No.: ______________________

Qualifying/Placement Exam, Part-A
10:00 – 12:00, January 9, 2018, 1400 BPS

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

| | |
|---|---|
| $k_e = 8.99\times10^9\ \text{Nm}^2/\text{C}^2$ | permittivity of free space |
| $\sigma = 5.7\times10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$ | Stefan-Boltzmann constant |
| $k = 1.4\times10^{-23}\ \text{J/K}$ | Boltzmann constant |
| $\hbar = 1.05\times10^{-34}\ \text{J}\cdot\text{s}$ | Planck's constant |
| $= 6.58\times10^{-16}\ \text{eV}\cdot\text{s}$ | " |
| $c = 3.0\times10^8\ \text{m/s}$ | speed of light |
| $e = 1.602\times10^{-19}\text{C}$ | charge of the electron |

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1. [10 pts] In a conservative system, consider the classical dynamics of a single particle of mass $m$, which is not subject to forces of constraints. The Lagrange function in Cartesian coordinates is, $L = T - U$, where $T$ is the Kinetic Energy $= \sum_i \frac{1}{2} m\dot{x}_i^2$, and $U$ is the Potential Energy, a function only of $(x_1, x_2, x_3)$. The Lagrangian equations of motion are:

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = 0, \quad i = 1,2,3.$$

Show that the Lagrangian equations of motion are equivalent to the Newtonian equations of motion.

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2. [10 pts] If one or both of these operations is identically zero, provide a proof below:

a) [5 pts] $\nabla \times \nabla \phi$, assuming a scalar field $\phi(x, y, z)$,

b) [5 pts] $\nabla \cdot \nabla \times \mathbf{A}$, assuming a vector field $\mathbf{A}(x, y, z)$.

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3. [10 pts] Use contour integration to solve the following integral:

$$I = \int_0^{2\pi} d\phi \frac{e^{i\phi}}{1-2e^{i\phi}} \ .$$

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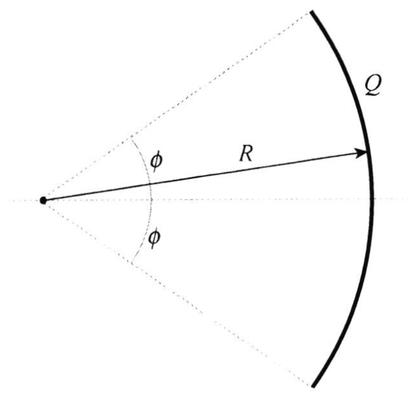
4. [10 pts] A charge $Q$ is distributed evenly on a wire bent into an arc of radius $R$ as shown, with an angle $\phi$ on either side of the bisector.




a) [7 pts] In the plane of the wire, determine the electric field at the center of the arc as a function of the angle $\phi$.

b) [3 pts] Sketch a graph of this electric field (at the center of the arc) as a function of $\phi$, the extent of the wire, for $0 < \phi < \pi$. Be sure to label the axes on the figure.

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5. [10 pts] A grounded ($V = 0$) conducting plane of infinite extent exists at $z = 0$. A point charge $q$ is brought in and placed a distance $d$ from the plane. What is the force (magnitude **and** direction) acting on the charge $q$?

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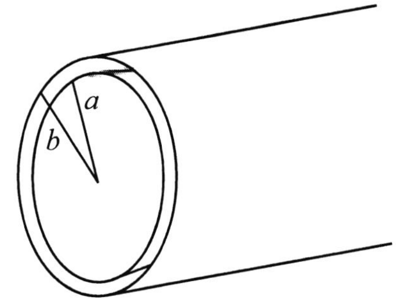


6. [10 pts] A variable capacitor consists of two thin and coaxial cylinders of radii, $a$ and $b$, ($b-a \ll a$) and of equal length, $L$, that are free to move with respect to each other only in the axial direction. Assume that the tubes have the ends initially aligned, are charged by a battery to a potential difference, $V_0$, and then disconnected from the battery.

a) [8 pts] Using energy methods, compute the magnitude and direction of the force on the inner cylinder when it is displaced outward by a small distance compared to the length of the tubes. Hint: In the region between the cylinders the magnitude of the electric field is essentially constant.

b) [2 pts] Explain how this force arises.