

Student No.: \_\_\_\_\_

Qualifying/Placement Exam, Part-A  
10:00 – 12:00, January 9, 2018, 1400 BPS

Put your **Student Number** on every sheet of this  
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose!*

You may need the following constants:

$k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$	permittivity of free space
$\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$	Stefan-Boltzmann constant
$k = 1.4 \times 10^{-23} \text{ J/K}$	Boltzmann constant
$\hbar = 1.05 \times 10^{-34} \text{ J}\cdot\text{s}$	Planck's constant
$= 6.58 \times 10^{-16} \text{ eV}\cdot\text{s}$	"
$c = 3.0 \times 10^8 \text{ m/s}$	speed of light
$e = 1.602 \times 10^{-19} \text{ C}$	charge of the electron

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1. [10 pts] In a conservative system, consider the classical dynamics of a single particle of mass  $m$ , which is not subject to forces of constraints. The Lagrange function in Cartesian coordinates is,  $L = T - U$ , where  $T$  is the Kinetic Energy  $= \sum_i \frac{1}{2} m \dot{x}_i^2$ , and  $U$  is the Potential Energy, a

function only of  $(x_1, x_2, x_3)$ . The Lagrangian equations of motion are:

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} = 0, \quad i = 1, 2, 3.$$

Show that the Lagrangian equations of motion are equivalent to the Newtonian equations of motion.

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2. [10 pts] If one or both of these operations is identically zero, provide a proof below:

a) [5 pts]  $\nabla \times \nabla \phi$ , assuming a scalar field  $\phi(x, y, z)$ ,

b) [5 pts]  $\nabla \cdot \nabla \times \mathbf{A}$ , assuming a vector field  $\mathbf{A}(x, y, z)$ .

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3. [10 pts] Use contour integration to solve the following integral:

$$I = \int_0^{2\pi} d\phi \frac{e^{i\phi}}{1-2e^{i\phi}} .$$

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4. [10 pts] A charge  $Q$  is distributed evenly on a wire bent into an arc of radius  $R$  as shown, with an angle  $\phi$  on either side of the bisector.

a) [7 pts] In the plane of the wire, determine the electric field at the center of the arc as a function of the angle  $\phi$ .

b) [3 pts] Sketch a graph of this electric field (at the center of the arc) as a function of  $\phi$ , the extent of the wire, for  $0 < \phi < \pi$ . Be sure to label the axes on the figure.

