Student No.: ______________________

Qualifying/Placement Exam, Part-B
2:00 – 4:00, January 9, 2018, 3239 BPS

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99 \times 10^9\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4 \times 10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05 \times 10^{-34}\ \mathrm{J \cdot s}$	Planck's constant
$= 6.58 \times 10^{-16}\ \mathrm{eV \cdot s}$	"
$c = 3.0 \times 10^8\ \mathrm{m/s}$	speed of light
$e = 1.602 \times 10^{-19}\mathrm{C}$	charge of the electron
$N_A = 6.022 \times 10^{23}\ \mathrm{mol^{-1}}$	Avogadro constant

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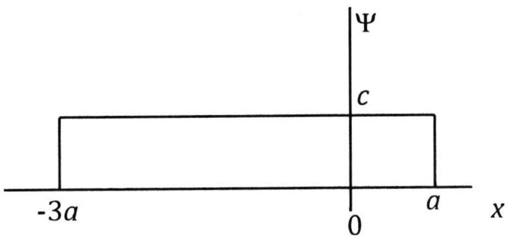
1. [10 pts] The state of a free particle is described by the following wave function



$$\psi(x) = 0 \text{ for } x < -3a$$
$$= c \text{ for } -3a < x < a \backslash$$
$$= 0 \text{ for } x > a$$

a) [2pts] Determine c using the normalization condition

b) [2pts] Show the calculation of the probability of finding the particle in the interval [0,a], and state why this makes sense.

c) [4pts] Compute $<x>$ and σ^2

d) [2pts] Calculate the momentum probability density

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2. [10 pts] The wave function for an electron in the 1s state of the hydrogen atom with the Coulomb potential, $V(r) = -e^2 / r$, has the form $\psi(\mathbf{x}) = C\exp(-\kappa r)$.

a) [4 pts] Show that this wave function is a solution to the time-independent Schroedinger equation for a certain value of κ. (Note: In the Schroedinger equation the wave function normalization constant C is a common factor)

b) [2 pts] Determine the value of κ in terms of the constants, $(e,\hbar,m_e,c)$, by matching the r dependence the terms.

c) [4 pts] From the results of part a), determine the energy of the electron in terms of those fundamental constants. (Note: the value of the fine-structure constant $e^2/\hbar c$ will be handy).

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3. [10 pts] The operators P and Q commute and are represented by the matrices:

$$\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \text{ and } \mathbf{Q} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}.$$

Find the eigenvalues and eigenvectors of both operators

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4. [10 pts] A thermal neutron at temperature, T=300 K, has a kinetic energy given by the equipartition theorem for an ideal gas.

a) [7 pts] Calculate the deBroglie wavelength of such neutrons.

b) [3 pts] Use the answer to part a) to determine if a beam of these neutrons can or cannot be diffractively scattered by a typical crystal