Statistical Mechanics Subject Exam, Monday, Dec. 11, 2017

SECRET STUDENT NUMBER: STUDNUMBER

DO NOT WRITE YOUR NAME OR STUDENT NUMBER ON ANY SHEET!

FUN FACTS TO KNOW AND TELL

$$\int_0^\infty dx\, \frac{x^{n-1}}{e^x - 1} = \Gamma(n)\zeta(n), \qquad \int_0^\infty dx\, \frac{x^{n-1}}{e^x + 1} = \Gamma(n)\zeta(n)\left[1 - (1/2)^{n-1}\right],$$

$$\zeta(n) \equiv \sum_{m=1}^{\infty} m^{-n}, \quad \Gamma(n) \equiv (n-1)!,$$

$$\zeta(3/2) = 2.612375..., \quad \zeta(2) = \frac{\pi^2}{6}, \quad \zeta(3) = 1.20205..., \quad \zeta(4) = \frac{\pi^4}{90},$$

$$\int_{-\infty}^{\infty} dx\, e^{-x^2/2} = \sqrt{2\pi}, \qquad \int_0^\infty dx\, x^n e^{-x} = n!$$

SECRET STUDENT NUMBER: STUDNUMBER

LONG ANSWER SECTION

1. (10 pts) Beginning with:

$$dE = TdS - PdV + \mu dN,$$

derive the Maxwell relation,

$$\left.\frac{\partial V}{\partial \mu}\right|_{S,P} = -\left.\frac{\partial N}{\partial P}\right|_{S,\mu}.$$

SECRET STUDENT NUMBER: STUDNUMBER

Extra workspace for #1

2. Consider the equation of state,

$$P = \rho T e^{\rho/\rho_0} - a\rho^2/\rho_0,$$

where ρ is the number density, T is the temperature, and ρ_0 and a are both positive constants.

(a) (10 pts) Find the critical density and critical temperature related to the first-order phase transition.

(b) (5 pts) What is the liquid density as $T \to 0$?

SECRET STUDENT NUMBER: STUDNUMBER

Extra workspace for #2

3. Consider a two-dimensional world (atoms move ONLY in the xy plane), where there is a two-dimensional metal where the longitudinal and transverse speeds of sound are both c_s. The material has spin 1/2 electrons of mass m_e and the density of electrons per area is ρ_e. The system is at a low temperature T. The temperature is much less than the Debye temperature and much less than the Fermi energy.

   (a) (10 pts) Find the contribution to the specific heat per area, $C_v^{(s)} = (1/A)dE/dT$, due to phonons.

   (b) (10 pts) Find the contribution to the specific heat per area, $C_V^{(f)}$, due to the Fermi motion of the electrons.

   Answers should be expressed in terms of T, c_s, m_e and ρ_e.