Student No.: ______________________

Qualifying/Placement Exam, Part-A
10:00 – 12:00, August 19, 2019

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	electric force constant
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{ev\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\ \mathrm{C}$	charge of the electron

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1. In polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$.

   a) [4 pts] Show that $\dfrac{\partial r}{\partial x} = \cos\theta$, $\dfrac{\partial r}{\partial y} = \sin\theta$, $\dfrac{\partial \theta}{\partial x} = \dfrac{-\sin\theta}{r}$, and $\dfrac{\partial \theta}{\partial y} = \dfrac{\cos\theta}{r}$.

   b) [6 pts] For a general function $U(x,y)$, express $\dfrac{\partial U}{\partial x}$ and $\dfrac{\partial U}{\partial y}$ in terms of the derivatives with respect to the polar coordinates, $\dfrac{\partial U}{\partial r}$ and $\dfrac{\partial U}{\partial \theta}$.

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2. Consider the following matrix,

$$\mathbf{M} = \begin{pmatrix} \alpha & \beta \\ \beta & \gamma \end{pmatrix}$$

   a) [5 pts] Find the eigenvalues of $\mathbf{M}$ for arbitrary real α, β and γ.
   b) [5 pts] If $\alpha = \beta = \gamma = 1$, find the normalized eigenvectors.

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3. Consider the periodic function

$$f(x) = \begin{cases} 0, & -L/2 \le x < 0 \\ F_0, & 0 \le x < L/2 \end{cases}$$

where F_0 is a constant and $f(x+mL) = f(x)$ for all positive and negative integers m.

If $f(x)$ is expressed as a Fourier series,

$$f(x) = \sum_{n=0,1,2\cdots} a_n \cos(2\pi nx/L) \quad + \sum_{n=1,2\cdots} b_n \sin(2\pi nx/L),$$

a) [2 pts] For which value(s) of n is a_n non-zero?
b) [2 pts] For which value(s) of n is b_n non-zero?
c) [6 pts] Find all non-zero values of a_n and b_n.

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4. A coaxial wire consists of a solid copper core of radius $a = 1.00$ mm surrounded by a copper sheath of inside radius $b = 1.50$ mm and outside radius $c = 2.00$ mm. A current, i, flows in one direction in the core and in the opposite direction in the sheath. The distance from the center of the wire is r.
   a) [2 pts] Derive the equation for the magnetic field as a function of r for $r < a$.
   b) [2 pts] Derive the equation for the magnetic field as a function of r for $a < r < b$.
   c) [2 pts] Derive the equation for the magnetic field as a function of r for $b < r < c$.
   d) [2 pts] Derive the equation for the magnetic field as a function of r for $r > c$.
   e) [2 pts] Graph the magnitude of the magnetic field in units of $\frac{\mu_0 i}{2\pi a}$ as a function of r.

5. An infinite conducting plane at $z = 0$ is kept at zero potential. A long wire of charge-per-unit length λ is brought in parallel to the plane at $z = a$ and aligned along the $\hat{y}$ direction, and has $x = 0$ for all points on the wire. Here a is a positive constant.
   a) [5 pts] Find the electric field at every point along a line defined by $x = y = 0$ as a function of z. Be sure to give the field for both $z < 0$ and $z > 0$.
   b) [5 pts] What is the electric field adjacent to the plane ($z = 0^+$) as a function of x and y? Be sure to provide all three components of $\vec{E}$.

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6. An electromagnetic plane wave is passing through the vacuum. The components of the electric field are

$$E_x = 0$$
$$E_y = p\cos(qx + rt)$$
$$E_z = s\cos(qx + rt)$$

where p, q, r, s are constants.

   a) [2 pts] Find the condition that the constants p, q, r, s must satisfy to make the speed correct.
   b) [5 pts] Find the magnetic field.
   c) [3 pts] Find the Poynting vector.