PHY-831 STATISTICAL MECHANICS
Subject Exam, Fall 2014
Total $4 \times 20 + 5 \times 4 = 100$ points

PROBLEM 1. Assume that in the systems of identical particles *parastatistics* can exist which allow, for any single-particle orbital $|\lambda)$, occupation numbers $n_\lambda = 0, 1$ and 2. Derive general expressions for the grand canonical thermodynamic potential Ω, mean occupancy $\bar{n}_\lambda$, and total energy E of such a system at temperature T. Describe the distribution of particles at $T \to 0$.

PROBLEM 2. Calculate the mean square fluctuation of the velocity of the particle in the ideal classical gas at temperature T.

PROBLEM 3. Consider a classical gas of electrons with applied electric field, $\mathcal{E}_x$, and gradient of temperature, dT/dx, along the same axis.

a. Find the electric current J_e and the heat flux J_Q using the kinetic Boltzmann equation in the approximation of constant relaxation time assuming the linear dependence of currents on electric field and temperature gradient. The heat flux is defined as $J_Q = \int d^3v\, \epsilon v_x f$, where ϵ is the particle energy, while the distribution function f is the solution of the kinetic equation.

b. The heat conductivity coefficient K is usually defined and measured through $J_Q = -K(dT/dx)$ under the condition of the absence of the electric current, $J_e = 0$. Find the coefficient K in the same model.

PROBLEM 4. *a*. Find the amount of work performed in the process of the isothermal expansion of the classical perfect gas from the volume V_1 to the volume V_2.

b. The same for the van der Waals gas.

QUICK QUESTIONS

1. The nuclei of two isotopes of the same chemical element have different ground state spins, $I = 0$ and $I \neq 0$. Compare energy E, free energy F, entropy S and heat capacity C_V of corresponding monatomic gases (N atoms at temperature T which is much lower than energies of nuclear excitations so all nuclei can be considered to be in their ground states). Neglect hyperfine interactions of nuclear spins with the electron angular momentum.

2. Treating CO_2 gas as a classical gas of linear molecules, find the specific heat (heat capacity C_V per particle) at high temperature (but still much lower than the energy of electronic excitations).

3. Estimate at what temperature the density of thermal radiation energy in equilibrium with a simple classical gas will be equal to the energy density of the gas itself.

4. Estimate the critical temperature of Bose-condensation for a gas of atoms of rubidium-87 at density $n \sim 10^{12}\,\mathrm{cm}^{-3}$. Is it possible to observe a similar effect for the isotope rubidium-86?

5. The line on the phase diagram (P, T) for ^{4}He that corresponds to the equilibrium between superfluid liquid helium and solid helium is horizontal at low temperature. What can we conclude from this fact about properties of those phases of helium?