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Theory of Nondegenerate Two-Dimensional Electrons Interacting with Phonons in Quantizing Magnetic Fields

By

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Under transverse magnetic fields the two-dimensional electron energy spectrum transforms into a set of essentially discrete Landau levels. Taking into account the non-parabolicity of the electron energy dispersion a quantum kinetic equation for the case of inelastic scattering by phonons is obtained. This equation is valid over a wide temperature range and at arbitrary ratios between the non-equidistance of the Landau levels (which is connected with both the non-parabolicity and the renormalization of the spectrum due to electron–phonon interaction) and their widths. The shape of the cyclotron resonance peak is considered in the deformation potential approximation and also in the case of scattering by phonons whose frequency is determined by their two-dimensional wave vector. In the latter case from the shape of the peak it is possible to obtain the phonon dispersion law.

В поперечном магнитном поле спектр энергии двумерных электронов представляет набор дискретных уровней Ландау. Получено квантовое кинетическое уравнение в случае неупругого рассеяния на фононах с учетом непараболичности дисперсии электрона. Уравнение справедливо в широком интервале температур и при произвольном соотношении между неэквидистантностью уровней Ландау (она связана с непараболичностью и с перенормировкой спектра, обусловленной взаимодействием с фононами) и их шириной. Рассмотрена форма пика циклотронного резонанса в приближении деформационного потенциала, а также при рассеянии на фононах, частота которых определяется их двумерным волновым вектором. В последнем случае по форме пика можно восстановить закон дисперсии фононов.

1. Introduction

The gas of carriers in size-quantized systems (e.g. thin solid films [1] or narrow surface layers in semiconductors [2]) is two-dimensional if the carrier concentration and temperature are limited, so that all two-dimensional sub-bands except the lowest one are empty. When the two-dimensional gas is inserted into a transverse magnetic field $\boldsymbol{H}$ the energy spectrum of the carriers (for the sake of concreteness we shall consider electrons) is transformed into a set of discrete Landau levels. Optical absorption due to the transitions of electrons between these levels (cyclotron resonance) was observed in the liquid He-gaseous He interface [3] and in the inversion layer of p-type Si [4 to 6].

The peculiar properties of entirely quantized electrons are connected with the discreteness of their energy spectrum. They are most clearly manifested at $\omega_c \tau \gg 1$, where ω_c is the cyclotron frequency and τ is the relaxation time. At $\omega_c \tau \gg 1$ the electron energy relaxation is due to essentially inelastic scattering by phonons (or to inelastic scattering by impurities; we shall consider pure systems and scattering by impurities will be neglected). During the transition of the electron between discrete Landau levels a phonon with corresponding fre-

quency is emitted or absorbed. In the elementary act the parallel (to the two-dimensional layer) momentum is conserved while the transverse momentum is transferred to the "walls" of the system. Such inelastic scattering differs qualitatively from the volume scattering by optical phonons where all components of the momentum are conserved. We shall assume that ω_c and its overtones do not coincide with singularities of the phonon density of states.

The dynamics of electrons depends strongly also on their dispersion law. The non-parabolicity of the dispersion leads to the non-equidistance of the discrete Landau levels. The non-equidistance is due also to electron–phonon interaction shifting the levels, the shift of different levels being different (two-dimensional magnetic polaron). In the following it is assumed that the non-equidistance of levels $\Delta\omega_c$ and the electron–phonon interaction are small so that $\Delta\omega_c, \tau^{-1} \ll \omega_c$.

In [7] it was shown that the line shape of cyclotron resonance (CR) of non-degenerate two-dimensional electrons at $T \sim \hbar\omega_c$ (there were electrons in a few Landau levels) depends strongly on the ratio $\Delta\omega_c/\tau^{-1}$. At $\Delta\omega_c \gg \tau^{-1}$ a fine structure appears in the spectrum near ω_c. Different lines of the fine structure correspond to transitions between different adjacent levels under the resonance electric field. The frequencies and the widths of the lines were determined in [7]. The fine structure of the CR spectrum is an attribute of two-dimensional systems. In the three-dimensional case the fine structure may be observed if the dispersion in a plane perpendicular to H is non-parabolic ($\Delta\omega_c \gg \tau^{-1}$), while the electron energy equals to the sum of energies of the transverse (to H) and longitudinal motion. With decreasing $\Delta\omega_c/\tau^{-1}$ different lines of the fine structure overlap and the CR spectrum becomes smooth (but asymmetric at $\Delta\omega_c \neq 0$).

In this work the dynamics of entirely quantized nondegenerate electrons is considered at arbitrary ratio $\Delta\omega_c/\tau^{-1}$ over a wide temperature range. In Section 2 the Green's function for the density matrix of electrons is introduced and using it the equation for the conductivity in the range of the CR peak is obtained (the equation for the two-particle Green's function is derived in the Appendix). In Section 3 the deformation potential approximation is analysed. In this approximation the conductivity is given by a comparatively simple expression. In Section 4 the shape of the CR spectrum in the case of scattering by phonons localized in a two-dimensional layer is investigated. Section 5 contains some concluding remarks.

2. Green's Function and Equation for Conductivity

For the sake of simplicity we shall consider an isotropic two-dimensional system. The results may be readily extended to the anisotropic case. The electron momentum and position vectors, p and $r = (x, y)$ and the vector potential $A = (0, Hx)$ $((Hr) = 0)$ are supposed to be two-dimensional. The phonon is specified by a two-dimensional wave vector q and by an index j which determines the branch number of the phonon, its polarization and the quantum number of its transverse motion.

The Hamiltonian of the electron–phonon system is of the form

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_i; \quad \mathscr{H}_0 = \frac{1}{2m} P^2 + \frac{V}{8m^2\omega_c^2} P^4 + \sum_{qj} \omega_{qj} a_{qj}^+ a_{qj}, \tag{1}$$

$$\mathscr{H}_i = \sum_{qj} \varepsilon_{qj} \exp\left(iqr\right) a_{qj}^+ + \text{c.c.}, \quad P = p - \frac{e}{c} A, \quad \hbar = 1. \tag{2}$$

Here m is the effective mass at the bottom of the band, $\omega_c = |e| H/mc$; V is the non-parabolicity parameter (in Kane's model $V = -2\omega_c^2/\varepsilon_g$). The interaction parameters ε_{qj} are assumed to be small so that $\omega_c \tau \gg 1$. The two-phonon interaction is investigated in the Appendix.

In (1) the electron spin is not taken into account. We shall assume $|g\mu_B H - \omega_c| \gg \tau^{-1}$ and consider electrons with fixed projection of spin (the electron ground state energy and, generally speaking, the cyclotron frequency are spin-dependent).

According to (1) the electron wave function is specified by the number n of the Landau level and the quasi-continuous number X corresponding to the x-projection of the centre of the cyclotron orbit. The energy of the electron located at the n-th level, E_n, does not depend on X, $E_n = \omega_c n + \frac{1}{2}Vn\,(n+1)$. In the following all traces will be calculated using electron wave functions $(nX|$ corresponding to $V = 0$ and phonon wave functions $(n_{qj}|$ in the occupation number representation.

The dynamics of an electron interacting with phonons is determined by its density matrix $\tilde{\varrho}$

$$\tilde{\varrho}(t) = Tr_{\text{ph}}\varrho(t) \equiv \sum_{\ldots,\,n_{qj},\ldots} (\ldots n_{qj} \ldots |\varrho(t)| \ldots n_{qj} \ldots) \,, \tag{3}$$

where $\varrho(t)$ is the density matrix of the whole system. Assuming that at $t = 0$ the phonons are in thermal equilibrium while the electron is described by $\tilde{\varrho}(0)$ we may introduce the two-particle Green's function $G(t)$:

$$(nX|\,\tilde{\varrho}(t)\,|n'X') = \exp\left[i(E_{n'} - E_n)\,t\right] \sum_{n_1, X_1, n_1', X_1'} G(n, X; n', X'|\,n_1, X_1; n_1', X_1'\,|t) \times$$
$$\times (n_1 X_1|\,\tilde{\varrho}(0)\,|n_1'X_1') \,. \tag{4}$$

It is of utmost interest to determine $G(t)$ in the large time range $t \gg \omega_c^{-1}$, $t \sim \tau$. This needs the use of asymptotic methods of perturbation theory. A convenient asymptotic method for the calculation of correlation functions of a nonlinear oscillator interacting with phonons was deduced in [8 to 10]. As the oscillator spectrum and the spectrum of a two-dimensional electron in a quantizing magnetic field are analogous (but the Landau levels are degenerate with respect to X), we succeeded in the generalization of the method [8 to 10] to the problem considered here. This allowed to obtain (see the Appendix) the differential-difference equation (A2) for the function $G(t)$. Using (A2) to (A4) one may easily find $G(t)$ for any concrete system with the aid of a computer.

The Green's function $G(t)$ allows to calculate any time correlation function and therefore to determine any transport coefficient of the nondegenerate isotropic electron gas. The correlation function of two arbitrary single-particle operators A and B with an accuracy to $(\tau^{-1}/T)\exp(-\omega_c/T) \ll 1$ equals

$$\langle A(t)\,B(0)\rangle = Z_e^{-1} \sum_{\substack{n, X, n', X', \\ n_1, X_1, n_1', X_1',}} (n'X'|A|nX)\,(n_1 X_1|B|n_1'X_1') \exp\left[i(E_{n'} - E_n)\,t\right] \times$$
$$\times \exp(-\lambda E_{n_1'})\,G(n, X; n', X'|\,n_1, X_1; n_1', X_1'\,|t)\,;$$
$$Z_e = \sum_{n, X} \exp(-\lambda E_n)\,, \qquad \lambda = 1/T \,. \tag{4a}$$

We shall consider the conductivity

$$\sigma(\omega) \equiv \sigma_{xx}(\omega) = N\,\frac{1 - \exp(-\lambda\omega)}{\omega}\,\text{Re} \int_0^{\infty} \langle j_x(t)\,j_x(0)\rangle\,e^{i\omega t}\,dt \,, \tag{5}$$

where N denotes the electron concentration and $\boldsymbol{j} \approx (e/m)\,\boldsymbol{P}$. Substituting $A = B = j_x$, $(nX|\,j_x\,|n'X') = e\,\sqrt{\omega_c/2m}\,\,\delta_{X,X'}[\delta_{n'+1,n}\,\sqrt{n} + \delta_{n'-1,n}\,\sqrt{n+1}]$ into (4a) we may write $\sigma(\omega)$ near the CR peak in the form

$$
\left.
\begin{aligned}
&\sigma(\omega) = \frac{e^2 N}{2m}\,Q(\omega); \quad \omega \approx \omega_c, \quad Q(\omega) = \operatorname{Re}\int_0^\infty Q(t)\,\exp\left[i(\omega - \omega_c)\,t\right]\mathrm{d}t; \\[2mm]
&Q(t) = (\bar{n}+1)^{-1}\sum_{n=0}^\infty \varphi(n,t); \quad \varphi(n,t) = Z_{\mathrm{e}}^{-1}\,\mathrm{e}^{-iV(n+1)t}\sum_{n',X,X'}\mathrm{e}^{-\lambda E_{n'}}\times \\[2mm]
&\times \sqrt{(n+1)(n'+1)}\,G(n+1,X;n,X|\,n'+1,X';n',X'\,|t)\,, \\[2mm]
&\bar{n} = [\exp(\lambda\omega_c) - 1]^{-1}\,.
\end{aligned}
\right\} \quad (6)
$$

The Fourier transform of $\varphi(n,t)$ may be compared with the optical absorption line due to the electron transition from the n-th Landau level to the $(n+1)$-th one. The whole spectrum $\sigma(\omega)$ presents the superposition of these lines according to (6). If in the case of electron–phonon scattering a phonon may be emitted only for an electron transition between adjacent Landau levels (i.e. if $k_{\mathrm{m}} = 1$, see the Appendix), then from (A7) the following equation may be obtained

$$
\left.
\begin{aligned}
\frac{\partial\varphi(n,t)}{\partial t} = &-[\Gamma(n) + iP(n)]\,\varphi(n,t) + 2(\bar{n}+1)(n+1)\,\Gamma(n,1)\times \\[2mm]
&\times \varphi(n+1,t) + 2\bar{n}(n+1)\,\Gamma(n-1,1)\,\varphi(n-1,t); \\[2mm]
\varphi(n,0) = &(\bar{n}+1)^{-1}(n+1)\,\exp(-\lambda\omega_c n)\,,
\end{aligned}
\right\} \quad (7)
$$

where (cf.(A7), (A4))

$$
\left.
\begin{aligned}
&\Gamma(n) = \sum_{\boldsymbol{q}}[\Gamma_0(\boldsymbol{q},n) + \Gamma_0(\boldsymbol{q},n+1)]\,, \\[2mm]
&P(n) = P_{n+1} - P_n + V(n+1)\,, \\[2mm]
&\Gamma(n,1) = \sum_{\boldsymbol{q}}\Gamma_1(\boldsymbol{q};n+1,n)\,[(n+1)(n+2)]^{-1/2}\,; \quad n \geqq 0\,, \\[2mm]
&\Gamma(-1,1) = 0\,.
\end{aligned}
\right\} \quad (8)
$$

It is easy to check up that $\sum_{\boldsymbol{q}}\Gamma_0(\boldsymbol{q},n)$ equals to the width of the n-th Landau level given by the Golden Rule. Within the framework of the Weisskopf-Wigner theory $2\Gamma(n)$ is the half-width of the line corresponding to the transition $|n\rangle \to |n+1\rangle$, while $\omega_c + P(n)$ is its frequency. The terms with $\Gamma(n,1)$ demonstrate the inapplicability of this theory to the CR spectrum in the general case. These terms lead to interference of the lines with different n, which is connected with approximate equidistance of the Landau levels (the non-equidistance being $\Delta\omega_c \ll \omega_c$). If the widths of the levels are small as compared with their non-equidistance,

$$
|P(n) - P(n \pm 1)| \gg \Gamma(n), |\Gamma(n,1)|\,, \tag{9}
$$

the lines with different n practically do not overlap and the CR spectrum has a distinct fine structure. In the same approximation for the renormalized electron spectrum as for the initial one, i.e. when

$$
P(n) \approx P + \tilde{V}n\,, \tag{10}
$$

the fine structure lines are equidistant with an accuracy to $\tilde{V}/\Gamma(n)$. The fine structure is considered in detail in [7] for arbitrary k_{m}.

The set of equations (7) allows to calculate the CR spectrum numerically for arbitrary ratio between the level widths and the non-equidistance and over a wide range of temperatures.

3. Deformation Potential Approximation

For the further analysis of the CR line shape it is necessary to choose a concrete model of electron–phonon interaction. The experiments both in two-dimensional [3 to 6] and three-dimensional cases are often carried out at $\omega_c \approx 10^{12}$ s^{-1}, i.e. when ω_c lies in the acoustic phonon frequency range. In this section we consider the scattering by acoustic phonons which are not localized in a two-dimensional layer (their wave number j is quasi-continuous). It is obvious from (A4) that the contribution of a phonon with wave vector $\boldsymbol{q}$ to $\Gamma_1(\boldsymbol{q})$ is proportional to exp $(-q^2l^2/2)$ i.e. the damping is due to phonons with $q \lesssim l^{-1}$. For typical values of $\omega_c = 1^{12}$ s^{-1}, $v_s = 5 \times 10^5$ cm/s (v_s denotes the velocity of sound) and $m = 10^{-28}$ g we have $l = \sqrt{\hbar/m\omega_c} \approx 3 \times 10^{-6}$ cm and the dimensionless parameter $\delta = v_s/l\omega_c \approx 0.2$. δ characterizes the contribution to the resonance acoustic phonon frequency $\omega_{qj} = \omega_c$ connected with $q \neq 0$. As δ is small the main contribution to damping is due to the phonons moving almost normally to the layer.

In the deformation potential approximation

$$\varepsilon_{\boldsymbol{q}j} = C_j[i(\boldsymbol{q}\boldsymbol{e}_{\boldsymbol{q}j}) + \alpha_j(\boldsymbol{e}_{\boldsymbol{q}j}\boldsymbol{e}_z)] \, , \tag{11}$$

where C_j specifies the deformation potential parameter, $\boldsymbol{e}_{\boldsymbol{q}j}$ is the phonon polarization vector, $\boldsymbol{e}_z$ is the unit vector normal to the layer and α_j is determined by the dependence of the phonon wave function on z (for plane waves $\alpha_j = ik_z$). For waves propagating almost normally to the layer, from (11)

$$q^{-1} \lim_{\Delta \to 0} \left(\Delta^{-1} \sum_{\substack{\boldsymbol{q}_1 \\ (q \leq q_1 \leq q + \Delta)}} |\varepsilon_{\boldsymbol{q}_1 j}|^2 \right) \approx \xi_j + \eta_j l^2 q^2, \quad \omega_{qj} \approx \omega_c \, , \tag{12}$$

where the terms of higher order in q are omitted while the term linear in q is absent. This is correct both for longitudinal waves $((\boldsymbol{e}_{\boldsymbol{q}j}\boldsymbol{e}_z) \approx 1 + a_j q^2)$ and transverse waves $((\boldsymbol{e}_{\boldsymbol{q}j}\boldsymbol{e}_z) \sim q, \; \xi_j = 0)$. In the vicinity of the two-dimensional layer the longitudinal and transverse waves are, generally speaking, mixed. However, (12) is valid for almost normal (to the layer) waves because their coefficient of mixing is proportional to q.

The frequencies of the considered resonance phonons equal

$$\omega_{qj} = \omega_{0j} + \delta^2 \, \omega_{0j}^{(1)} q^2 l^2 \; ; \qquad \omega_{qj} \approx \omega_c \, , \qquad \omega_{0j}^{(1)} \approx \omega_c \, . \tag{13}$$

The second addend on the right of (13) may be neglected because $\delta^2 \ll 1$. In the model under consideration ω_{0j} is continuous near ω_c.

In the approximation (12) and (13) the summation over $\boldsymbol{q}$ in (8) may be fulfilled (cf. [11], 7.414.4). The resulting expressions for the damping parameters are

$$\Gamma(n) = \gamma_0 \left[2\bar{n} + 1 - \frac{1}{2} \delta_{n,0}(\bar{n} + 1) \right] + 2\gamma_1 [2(n + 1)(2\bar{n} + 1) - 1],$$

$$\Gamma(n, 1) = -\gamma_1;$$

$$\gamma_0 = \frac{2\pi}{l^2} \sum_j \xi_j \, \delta \, (\omega_{0j} - \omega_c), \quad \gamma_1 = \frac{2\pi}{l^2} \sum_j \eta_j \delta \, (\omega_{0j} - \omega_c).$$

$$\left. \right\} \tag{14}$$

It is seen from (6), (7), and (14) that the terms ξ_j and η_j in (12) lead to different effects. At $\eta_j = 0$ the CR spectrum presents a set of Lorentzian lines with half-widths $2\gamma_0 \left[2\bar{n} + 1 - (\frac{1}{2})\, \delta_{n,0}(\bar{n} + 1)\right]$, the intensity of the n-th line being proportional to $(n + 1) \exp(-\lambda\omega_c n)$. If the distance between the lines exceeds their widths the fine structure appears in a certain temperature range. In the opposite case the spectrum is smooth and asymmetric. At $\bar{n} = 0$ it becomes Lorentzian.

At $\gamma_1 \neq 0$ the individual lines $\varphi(n, \omega)$ cease to be Lorentzian, generally speaking. If the fine structure is distinct, the linewidth increases monotonously (approximately linearly) with rising number of lines. The set of equations (7), (14) may be solved at $\gamma_0 = 0$ in the approximation (10) using the method of generating functions (cf. [8, 10]). Then

$$
\begin{aligned}
Q(t) &= \left\{ \operatorname{ch} at + \left[\frac{i\tilde{V}}{4\gamma_1}(2\bar{n} + 1) + 6\bar{n}^2 + 6\bar{n} + 1\right] \frac{2\gamma_1}{a} \operatorname{sh} at \right\}^{-2} \times \\
&\quad \times \exp\left[(2\gamma_1 - iP + i\tilde{V})\, t\right] ; \\
&t \gg \omega_c^{-1}, \quad a^2 = 4\gamma_1^2 \left[\left(2\bar{n} + 1 + \frac{i\tilde{V}}{4\gamma_1}\right)^2 - \bar{n}(\bar{n} + 1)\right].
\end{aligned}
\tag{15}
$$

Equations (15) and (6) express the correlator of the electric current in elementary functions while the conductivity in quadratures for arbitrary $\tilde{V}/\gamma_1$. It is seen that at $\bar{n} \neq 0$ the correlator damping is non-exponential and hence the peak of $\sigma(\omega)$ is non-Lorentzian (but symmetric at $\tilde{V} = 0$). The asymmetry of $\sigma(\omega)$ increases with the increase in $\tilde{V}/\gamma_1$ and when (9) is fulfilled a fine structure appears in the spectrum. The shape of $\sigma(\omega)$ at $\gamma_0 = 0$ is shown in Fig. 1 for several values of $\tilde{V}/\gamma_1$ and $\bar{n}$. The curves are obtained 'by both numerical solution of the set of difference equations for $\varphi(n, \omega)$, corresponding to (7) ((10), (14) being taken into account), and integration of (15). The strong broadening of $\sigma(\omega)$ with increasing temperature is typical at $\tilde{V} = 0$. The fine structure is distinct at large $|\tilde{V}|/\gamma_1$ $(|\tilde{V}|/2\gamma_1 \gtrsim 10)$. It rapidly smears with temperature increase.
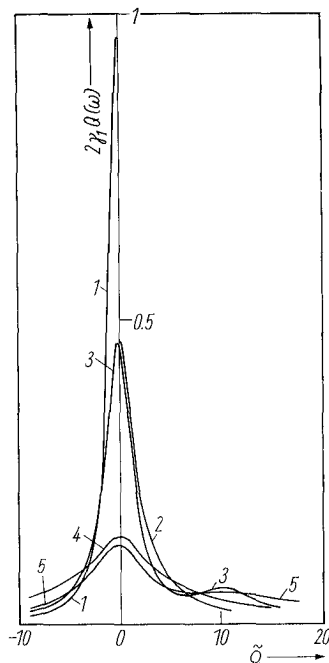

Fig. 1. Broadening of the cyclotron resonance line and the smearing of its fine structure with increasing temperature in the deformation potential approximation ($\xi_j = 0$), $\tilde{\Omega} = (\omega - \omega_c - P)/2\gamma_1$. The curves 1 to 5 correspond to the parameter values (1) $\omega_c/T = 5$, $V = 0$; (2) $\omega_c/T = 2$, $\tilde{V} = 0$; (3) $\omega_c/T = 2$, $\tilde{V}/2\gamma_1 = 10$; (4) $\omega_c/T = 1$, $\tilde{V} = 0$; (5) $\omega_c/T = 1$, $V/2\gamma_1 = 10$. The figure shows the halves of the symmetrical curves 1 and 2

4. Scattering by Two-Dimensional Phonons

The interaction with phonons localized in the thin layer occupied by electrons will be considered here. Rayleigh waves at a surface of solid serve as an example of such two-dimensional modes. These modes appear also in thin films with fixed or free boundaries where the spectrum of three-dimensional phonons is split into a set of two-dimensional sub-bands. The two-dimensional phonon frequency ω_{qj} within the limits of a sub-band (the sub-bands are numbered by the discrete quantity j) is determined by q and generally speaking, $\omega_{0j} \neq 0$. The condition of energy conservation in the case of electron–phonon scattering, $\omega_{qj} = \omega_c$, determines therefore the wave vector q_r of the resonance phonon. We shall denote

$$2 \lim_{\Delta \to 0} \left(\Delta^{-1} \sum_{\substack{q \\ (q_r \leq q \leq q_r + \Delta)}} \Gamma_1(q) \right) = \Gamma_r , \qquad x_r = q_r l . \tag{16}$$

Γ_r, x_r, and T/ω_c determine the damping parameters $\Gamma(n)$, $\Gamma(n, 1)$. Since $\Gamma_r \sim \exp\left(-x_r^2/2\right)$ (see (A4)) the scattering by two-dimensional phonons is inessential at $x_r \gg 1$.

Using (8), (10), and (16) the set of linear equations for $\varphi(n, \omega)$, corresponding to (7) may be solved numerically for arbitrary $\tilde{V}/\Gamma_r$, x_r, $\bar{n}$. The results of the calculation are shown in Fig. 2 and 3 for a few values of these parameters. Analytically the resonance scattering by the phonons with relatively long wavelength ($x_r \ll 1$, but $x_r \neq 0$ because at $x_r = 0$ $\Gamma_r = 0$) may be investigated. For $x_r \ll 1$ $L_{q_r}(n, 1) \approx \sqrt{2(n + 1)}$ (cf. [11], 8.970.1) and (7) coincides with equations in [8] which describe the interference of the absorption lines corresponding to transitions between the levels of an anharmonic oscillator interacting with phonons.[1]) The method of generating functions [8] gives

$$\left.\begin{aligned}
Q(t) &= \exp\left[(\Gamma_r - iP + i\tilde{V}) t\right] \left\{ \mathrm{ch}\,(\tilde{a}t) + \frac{\Gamma_r}{\tilde{a}}\left[1 + i\frac{\tilde{V}}{2\Gamma_r}(2\bar{n} + 1)\right] \mathrm{sh}\,(\tilde{a}t) \right\}^{-2} ; \\
x_r &\ll 1, \qquad \tilde{a}^2 = \Gamma_r^2 + i\Gamma_r \tilde{V}(2\bar{n} + 1) - \frac{\tilde{V}^2}{4}.
\end{aligned}\right\} \tag{17}$$

According to (17), (6) the shape of the CR peak is determined by an integral with elementary functions for arbitrary ratio between the non-equidistance of the Landau levels and their width Γ_r and for arbitrary temperature. For $\tilde{V} = 0$ $Q(t) = \exp\left[-(\Gamma_r + iP) t\right]$ and the shape of $\sigma(\omega)$ is Lorentzian with the half-width $2\Gamma_r$. As $|\tilde{V}|/\Gamma_r$ grows the asymmetry of peak increases (as $\tilde{V}^3/\Gamma_r^3$ at $|\tilde{V}| \ll \Gamma_r$) and at $|\tilde{V}| \gg \Gamma_r(2\bar{n} + 1)$ the fine structure appears. The half-widths $2\Gamma(n)$ of the lines of the fine structure increase linearly with rising number of line.

The terms of higher order in x_r lead to a non-Lorentzian shape of $\sigma(\omega)$ even for $\tilde{V} = 0$. For $|\tilde{V}|/\Gamma_r < 1$, $x_r < 1$ equation (7) may be solved by decoupling the chain of equations for the moments $Q_m(t) = \sum_{n=0}^{\infty} n^m \varphi(n, t)$. To the first order

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[1]) This coincidence is not accidental. In the investigation of light absorption by the oscillator it was assumed in [8] that the scattering by a phonon does not shift the oscillator equilibrium position. For $q_r l \ll 1$ the centre of electron cyclotron orbit is practically not shifted during inelastic scattering, therefore the quantized electron is analogous to the oscillator.

in x_{r}, $|\tilde{V}|/\Gamma_{\mathrm{r}}$ this yields after Fourier transformation

$$Q(\omega) = \frac{\Gamma_{\mathrm{r}}}{\Gamma_{\mathrm{r}}^2 + \tilde{\Omega}^2}\left(1 + 2\bar{n}x_{\mathrm{r}}\frac{\Gamma_{\mathrm{r}}^2 - \tilde{\Omega}^2}{\Gamma_{\mathrm{r}}^2 + \tilde{\Omega}^2}\right), \quad \tilde{\Omega} = \omega - \omega_{\mathrm{c}} - P - 2\tilde{V}\bar{n}. \quad (18)$$

According to (18) the peak remains symmetric for small x_{r} to first order in $\tilde{V}/\Gamma_{\mathrm{r}}$. It is interesting that the peak narrows and its maximum value increases with increasing temperature.

At $x_{\mathrm{r}} \approx 1$ $\Gamma(n)$ depends nonmonotonously on n and for some n the ratio $\Gamma(n)/\Gamma_{\mathrm{r}}$ may be small. This softens the criterion of the observation of a fine structure $|\tilde{V}| \gg \Gamma_{\mathrm{r}}$. The fine structure is clearly manifested even at $\tilde{V} = \Gamma_{\mathrm{r}}$ (see Fig. 2). At $x_{\mathrm{r}} \approx 1$ the shape of the spectrum is rather complicated, the amplitudes and widths of fine structure lines depend nonmonotonously on the number of lines and vary with temperature substantially (at $T \ll \hbar\omega_{\mathrm{c}}$ the spectrum presents a Lorentzian peak with the half-width $2\Gamma_{\mathrm{r}}$). For $\tilde{V} = \Gamma_{\mathrm{r}}$ only a part of lines of the fine structure may be seen while the rest lines overlap. At higher nonequidistance, when $\tilde{V}/\Gamma_{\mathrm{r}} = 10$, all lines are resolved practically if the temperature is not too high.

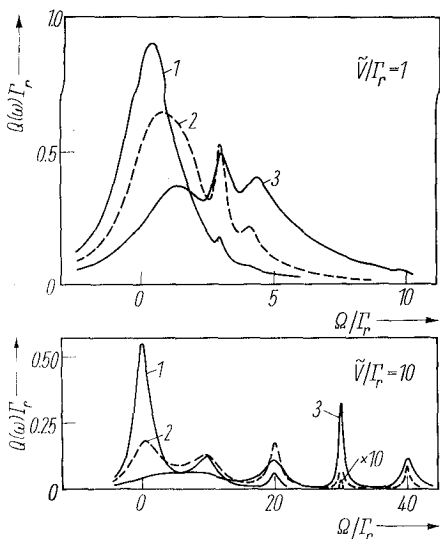

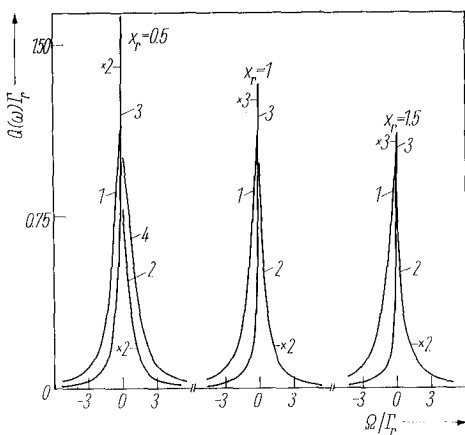Fig. 2   Fig. 3

Fig. 2. Shape of the cyclotron resonance line in the case of scattering by two-dimensiona phonons for $x_{\mathrm{r}} = 1$; $\tilde{V}/\Gamma_{\mathrm{r}} = 1$ and 10. $\Omega = \omega - \omega_{\mathrm{c}} - P$. The curves 1 to 3 correspond to the temperature values $\omega_{\mathrm{c}}/T = 2$; 1; 0.5

Fig. 3. Shape of the cyclotron resonance line in the case of scattering by two-dimensional phonons for $\tilde{V} = 0$ and $x_{\mathrm{r}} = 0.5$; 1; 1.5. $\Omega = \omega - \omega_{\mathrm{c}} - P$. The curves 1 to 4 correspond to the temperature values $\omega_{\mathrm{c}}/T = 2$; 1; 0.5; 5. The shape of the curve 4 is practically Lorentzian and does not depend on x_{r}. The right or left halves of the symmetrical curves are shown

The non-monotony of $\Gamma(n)$, $\Gamma(n, 1)$ causes another interesting effect. As can be seen from Fig. 3, for $\tilde{V} = 0$ and $x_{\rm r} \approx 1$ over a rather wide temperature range the rise in temperature leads to the narrowing of the peak of $\sigma(\omega)$ (for $\tilde{V} = 0$ all lines $\varphi(n, \omega)$ in (6) have a common peak and cannot be identified[2])).

The non-monotony of $\Gamma(n)$, $\Gamma(n, 1)$ appears to be essential at rather small $x_{\rm r}$ (e.g. at $x_{\rm r} = 0.5$ and $\tilde{V} = \Gamma_{\rm r}$ a few lines of the fine structure are distinguishable at $T/\omega_{\rm c} \approx 1$). The shape of the spectrum and its temperature dependence are very sensitive to $x_{\rm r}$, especially at $|\tilde{V}| \gg \Gamma_{\rm r}$, therefore the solution of the inverse problem (i.e. the determination of $x_{\rm r}$, $\tilde{V}$, and $\Gamma_{\rm r}$ from the experimental spectra) may be easily found using (6) to (8) and (16). The experimental determination of $x_{\rm r}$ for different $\omega_{\rm c}$ gives the dispersion of the two-dimensional phonons directly.

5. Conclusion

The peculiarity of the dynamics of two-dimensional electrons in quantizing magnetic fields is due to the discreteness of their energy spectrum. The evolution of the electron density matrix is determined by the ratio of non-equidistance of the Landau levels to their width. If the non-equidistance is relatively large, each pure electron state is damping exponentially with its lifetime. When the non-equidistance is of the same order as the width, all electron states interfere. This interference is described by the differential-difference equation (A2) for the two-particle Green's function. Equation (A2) is valid for both one- and two-phonon interaction.

Many properties of quantized electrons may be investigated using CR. The CR absorption peak has a complex asymmetric form, a fine structure being pronounced sometimes. The shape of the peak allows to determine the electron energy spectrum as well as the strength and type of electron–phonon interaction and the type of phonons causing inelastic scattering.

In the case of scattering by non-localized acoustic phonons (probably it takes place in quantized surface layers) in the deformation potential approximation the shape of the peak appears to be non-Lorentzian at finite temperature and depends strongly on the polarization of the scattering phonons. The increase in temperature causes rapid line broadening .

In the case of inelastic scattering by two-dimensional phonons (it is of importance in thin films with free or fixed boundaries) the shape of the CR spectrum depends strongly on the momentum of the resonance phonon. This allows to obtain the dispersion of phonons from experimental data. Increase in temperature may lead to narrowing of the CR line. The criterion for the observation of the fine structure (being determined by the ratio of the non-equidistance to the low-temperature half-width of the line) is substantially softer here than in the case of scattering by non-localized acoustic phonons.

The energy spectrum of electrons and electron–phonon interaction may be determined most precisely by the investigation of the temperature dependence of the CR spectrum when a fine structure is distinct. As with the increasing

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[2]) This fact was noticed in [9] in connection with the "paradoxon" of the harmonic oscillator. The identification is possible, strictly speaking, only for $|\tilde{V}| \gg \Gamma_{\rm r}$, when all lines of the fine structure are resolved.

magnetic field strength the non-equidistance of the Landau levels increases, in crystals with non-parabolic dispersion the fine structure must appear under strong fields (cf. [7]).

Appendix

The Green's function $G(t)$ may be expressed using the operator $U(t) = \exp(i\mathcal{H}_0 t) \exp(-i\mathcal{H}t)$:

$$
\left.
\begin{aligned}
&G(n, X; n', X' \mid n_1, X_1; n_1', X_1' \mid t) = Z_{\mathrm{ph}}^{-1} \sum_{\ldots, n_{qj}, m_{qj}, \ldots} \exp\left(-\lambda \sum \omega_{qj}\, m_{qj}\right) \times \\
&\times (\ldots n_{qj} \ldots; nX \mid U(t) \mid \ldots m_{qj} \ldots; n_1 X_1) \times \\
&\times (\ldots m_{qj} \ldots; n_1'X_1' \mid U^{-1}(t) \mid \ldots n_{qj} \ldots; n'X'); \\
&Z_{\mathrm{ph}} = \prod_{qj} (\bar{n}_{qj} + 1), \quad \bar{n}_{qj} = [\exp(\lambda\omega_{qj}) - 1]^{-1}, \quad \lambda = \frac{1}{T}.
\end{aligned}
\right\} \quad (A1)
$$

The chronological ordering allows to carry out the summation over n_{qj}, m_{qj} in (A1). Then $G(t)$ is determined by the integrals (over time) of the ordered operators. The expressions under the integrals may be divided into smooth and fast oscillating (like $\exp(i\omega_c t)$) parts. The integral of the latter between the limits 0 and $t \gg \omega_c^{-1}$ is small (as compared with the integral of the former). Neglecting it we may obtain the equation for $G(t)$ (the analogous procedure in the case of an oscillator interacting with phonons is described in [8]):

$$
\left.
\begin{aligned}
&\frac{\partial G(n, X; n', X' \| t)}{\partial t} = \sum_{k=-k_m}^{k_m} \sum_{q} D_{kq}(n, n') \exp[-iVk(n-n')\,t] \times \\
&\times \exp[iq_x(X-X')\chi(k)]\, G(n+k, X - l^2 q_y \chi(k); n'+k, X' - l^2 q_y \chi(k) \| t); \\
&\chi(k) = \frac{k}{|k|}(1 - \delta_{k,0}), \quad l^2 = (\omega_c m)^{-1}, \\
&G(m, Y; m', Y' \| t) \equiv G(m, Y; m', Y' \mid n_1, X_1; n_1', X_1' \mid t), \\
&G(n, X; n', X' \| 0) = \delta_{n,n_1}\delta_{X,X_1}\delta_{n',n_1'}\delta_{X',X_1'},
\end{aligned}
\right\} \quad (A2)
$$

where

$$
\left.
\begin{aligned}
&D_{0q}(n, n') = -\Gamma_0(q, n) - \Gamma_0(q, n') - i[P(q, n) - P(q, n')], \\
&D_{kq}(n, n') = 2\Gamma_k\left(q; n + \frac{k}{2} - \frac{|k|}{2}, n' + \frac{k}{2} - \frac{|k|}{2}\right) \times \\
&\qquad\qquad \times \left(\bar{n}_k + \frac{1}{2} + \frac{1}{2}\chi(k)\right); \\
&k \neq 0, \quad \bar{n}_k = [\exp(\lambda\omega_c|k|) - 1)]^{-1}.
\end{aligned}
\right\} \quad (A3)
$$

The parameters Γ and P are quadratic in ε_{qj}:

$$\Gamma_0(q, n) = \sum_{k=1}^{k_m} \Gamma_k(q) \left[\bar{n}_k L_q^2(n, k) + (\bar{n}_k + 1) L_q^2(n - k, k) \right],$$

$$\Gamma_k(q; n, n') = \Gamma_k(q) L_q(n, |k|) L_q(n', |k|),$$

$$\Gamma_k(q) = \pi \sum_j |\varepsilon_{qj}|^2 \left(\frac{1}{2} lq \right)^{2|k|} \exp\left(-\frac{1}{2} l^2 q^2 \right) \delta(|k| \, \omega_c - \omega_{qj}),$$

$$P(q, n) = \text{v.p.} \sum_j |\varepsilon_{qj}|^2 \exp\left(-\frac{1}{2} l^2 q^2 \right) \sum_{m=-n}^{\infty} \left(\frac{1}{2} lq \right)^{2|m|} \times$$

$$\times \left[\frac{\bar{n}_{qj}}{\omega_{qj} - m\omega_c} - \frac{\bar{n}_{qj} + 1}{\omega_{qj} + m\omega_c} \right] L_q^2\left(n + \frac{m}{2} - \frac{|m|}{2}, |m| \right);$$

$$L_q(n, m) = L_n^m\left(\frac{1}{2} l^2 q^2 \right) \left[\frac{2^m n!}{(n + m)!} \right]^{1/2}, \quad n \geqq 0, \quad L_q(n, m) \equiv 0, \quad n < 0.$$

$$\left. \right\} \text{(A4)}$$

L_n^m are Legendre polynomials. They appear because

$$(nX| \exp(iqr) |n + m \, X - l^2 q_y) \sim L_q\left(n + \frac{m}{2} - \frac{|m|}{2}, |m| \right).$$

The corrections of the order of

$$\left| \sum_q D_{kq}/\omega_c \right|, \qquad |\partial(\sum_q D_{kq})/\partial\omega_c| \ll 1 \tag{A5}$$

are omitted in (A2). It is seen from (A2) that $G(t)$ falls off non-exponentially, therefore it is impossible to introduce a relaxation time τ. The condition (A5) substitutes for the condition $\omega_c \tau \gg 1$ used in three-dimensional problems and shows how small the electron-phonon interaction must be.

It is obvious from (4) that the density matrix $\tilde{\varrho}(t)$ in the interaction representation satisfies (A2) also. In effect, equation (A2) is a balance equation. The terms with $k = 0$ in the right-hand side of (A2) describe all electron transitions from the states $(nX|$ and $|n'X')$ due to emission or absorption of a phonon, as well as the shift of the energy of these states. The terms with $k \neq 0$ describe all transitions to the states $(nX|$ and $|n'X')$ from other states. The δ-functions in $\Gamma_k(q)$ in (A4) conform to the energy conservation for the real transitions. The n-th level shift,

$$P_n = \sum_q P(q, n), \tag{A6}$$

is due to virtual processes (weakly-bound two-dimensional magnetic polaron).

Since the matrix elements of some single-particle operators (e.g. momentum or energy of electron) are diagonal with respect to X and independent of X, the equation for the function $G(n, n' |t) = \sum_X G(n, X; n', X| n_1, X_1; n_1', X_1' |t)$ may be useful:

$$\frac{\partial G(n, n' |t)}{\partial t} = \sum_{k=-k_m}^{k_m} D_k(n, n') \exp\left[-iVk (n - n') t \right] G(n + k, n' + k |t);$$

$$D_k(n, n') = \sum_q D_{kq}(n, n'); \quad G(n, n' |0) = \delta_{n, n_1} \delta_{n', n_1'} \delta_{X_1, X_1'}.$$

$$\left. \right\} \text{(A7)}$$

k_m in (A2) and (A7) is the maximum difference in the numbers of such two levels that a phonon-assisted transition from one of them to another is possible. For high ω_c ($\gtrsim 10^{12}$ s^{-1}) used in experiments on CR in two-dimensional systems [3 to 6] k_m is small. In the case of interaction with acoustic phonons one may often assume $k_\mathrm{m} = 1$, i.e. take into account only phonon-assisted transitions between adjacent Landau levels. This is due to the restriction to the resonance phonon wave numbers: $q \lesssim l^{-1}$ because $\Gamma_k(\boldsymbol{q})$ is exponentially small at $ql \gg 1$ (see (A4)), while the transverse wave number must not exceed substantially the inverse thickness of the electron layer. Since the decays corresponding to $k = 1$ cause finite width of the CR line for $\omega_\mathrm{c}/T \gg 1$, the account of them is of principle importance. In case of scattering by optical phonons the condition $k_\mathrm{m} = 1$ is not obligatory. However, in this work for the sake of brevity the consideration of the shape of the CR spectrum is carried out at $k_\mathrm{m} = 1$. This approximation is often sufficient at high ω_c.

If the cyclotron frequency either lies in the gap between acoustic and optical branches or exceeds the optical phonon frequency the single-phonon transitions between Landau levels may be forbidden due to the energy conservation law (an analogous situation appears in the case of localized vibrations of impurities in crystals [12]). Then, similarly to the theory of localized vibrations, the two-phonon processes are to be taken into account. The Hamiltonian of the two-phonon interaction is of the form

$$\left. \begin{aligned} \mathscr{H}_\mathrm{i}^{(2)} &= \sum_{\boldsymbol{q}} \exp\left(i\boldsymbol{qr}\right) \sum_{\boldsymbol{q}_1 j j_1} (\varepsilon_{\boldsymbol{q}\boldsymbol{q}_1 j j_1} a^+_{\boldsymbol{q}_1 j} a^+_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} + \tilde{\varepsilon}_{\boldsymbol{q}\boldsymbol{q}_1 j j_1} a^+_{\boldsymbol{q}_1 j} a_{-\boldsymbol{q}+\boldsymbol{q}_1 j_1}) + \text{c.c.};\\ \varepsilon_{0\boldsymbol{q}_1 j j_1} &= \tilde{\varepsilon}_{0\boldsymbol{q}_1 j j_1} = 0 \, . \end{aligned} \right\} \quad (A8)$$

To the second order in $\mathscr{H}_\mathrm{i}^{(2)}$ equations (A2) to (A4) may be obtained again, where in $\Gamma_k(\boldsymbol{q})$ and $P(\boldsymbol{q}, n)$ new terms $\Gamma_k^{(2)}(\boldsymbol{q})$ and $P^{(2)}(\boldsymbol{q}, n)$ appear:

$$\left. \begin{aligned} \Gamma_k^{(2)}(\boldsymbol{q}) &= 2\pi \sum_{\boldsymbol{q}_1 j j_1} \left(\frac{1}{2} lq\right)^{2|k|} \exp\left(-\frac{1}{2} l^2 q^2\right) [|\varepsilon_{\boldsymbol{q}\boldsymbol{q}_1 j j_1}|^2 \, (\bar{n}_{\boldsymbol{q}_1 j} + \bar{n}_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} + 1) \times \\ &\quad \times \delta(\omega_{\boldsymbol{q}_1 j} + \omega_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} - |k| \, \omega_\mathrm{c}) + 2 \, |\tilde{\varepsilon}_{\boldsymbol{q}\boldsymbol{q}_1 j j_1}|^2 \, (\bar{n}_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} - \bar{n}_{\boldsymbol{q}_1 j}) \times \\ &\quad \times \delta(\omega_{\boldsymbol{q}_1 j} - \omega_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} - |k| \, \omega_\mathrm{c})] \, , \\[4pt] P^{(2)}(\boldsymbol{q}, n) &= 2 \, \text{v.p.} \sum_{\boldsymbol{q}_1 j j_1} \exp\left(-\frac{1}{2} l^2 q^2\right) \sum_{m=-n}^{\infty} \left(\frac{1}{2} lq\right)^{2|m|} \times \\ &\quad \times L_q^2\left(n + \frac{m}{2} - \frac{|m|}{2}, |m|\right) \left[|\varepsilon_{\boldsymbol{q}\boldsymbol{q}_1 j j_1}|^2 \left(\frac{\bar{n}_{\boldsymbol{q}_1 j} \bar{n}_{\boldsymbol{q}-\boldsymbol{q}_1 j_1}}{\omega_{\boldsymbol{q}_1 j} + \omega_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} - m\omega_\mathrm{c}} - \right. \right. \\ &\quad \left. - \frac{(\bar{n}_{\boldsymbol{q}_1 j} + 1)(\bar{n}_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} + 1)}{\omega_{\boldsymbol{q}_1 j} + \omega_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} + m\omega_\mathrm{c}}\right) + \\ &\quad \left. + 2 \, |\tilde{\varepsilon}_{\boldsymbol{q}\boldsymbol{q}_1 j j_1}|^2 \left(\frac{\bar{n}_{\boldsymbol{q}_1 j}(\bar{n}_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} + 1)}{\omega_{\boldsymbol{q}_1 j} - \omega_{\boldsymbol{q}-\boldsymbol{q}_1 j_1} - m\omega_\mathrm{c}}\right)\right] \, . \end{aligned} \right\} \quad (A9)$$

Besides there appear terms causing the addition of the expression $- \sum_{\boldsymbol{q}} \gamma_{\boldsymbol{q}}(n, n')$ to $D_0(n, n')$ in (A7),

$$\gamma_{\boldsymbol{q}}(n, n') = 4\pi \sum_{\boldsymbol{q}_1 j j_1} |\tilde{\varepsilon}_{\boldsymbol{q}\boldsymbol{q}_1 j j_1}|^2 \, \bar{n}_{\boldsymbol{q}_1 j}(\bar{n}_{\boldsymbol{q}_1 j} + 1) \exp\left(-\frac{1}{2} l^2 q^2\right) \times$$
$$\times [L_q(n, 0) - L_q(n', 0)]^2 \, \delta(\omega_{\boldsymbol{q}_1 j} - \omega_{\boldsymbol{q}-\boldsymbol{q}_1 j_1}) \, . \tag{A10}$$

This addend leads to a specific broadening of the Landau levels which is not connected with transitions between them. It corresponds to the elastic scattering of phonons by electrons and may dominate for large ω_c/ω_m (ω_m is the maximum frequency of phonons).

References

[1] M. I. ELINSON, V. A. VOLKOV, V. N. LUTSKIJ, and T. N. PINSKER, Thin Solid Films 12, 383 (1972).
[2] F. STERN and W. E. HOWARD, Phys. Rev. 163, 816 (1967).
[3] T. R. BROWN and C. C. GRIMES, Phys. Rev. Letters 29, 1233 (1972).
[4] G. ABSTREITER, P. KNESCHAUREK, J. P. KOTTHAUS, and J. F. KOCH, Phys. Rev. Letters 32, 104 (1974).
[5] S. J, ALLEN, JR., D. C. TSUI, and J. V. DALTON, Phys. Rev. Letters 32, 107 (1974).
[6] J. P. KOTTHAUS, G. ABSTREITER, and J. F. KOCH, Solid State Commun. 15, 517 (1974).
[7] M. I. DYKMAN, Fiz tverd. Tela 17, 3007 (1975).
[8] M. I. DYKMAN and M. A. KRIVOGLAZ, Zh. eksper. teor. Fiz. 64, 993 (1973).
[9] M. I. DYKMAN and M. A. KRIVOGLAZ, phys. stat. sol. (b) 68, 111 (1975).
[10] M. I. DYKMAN, Zh. eksper. teor. Fiz. 68, 2082 (1975).
[11] I. S. GRADSHTEYN and I. M. RYZHIK, Tablitsy integralov, sum, ryadov i proizvedenii, Izd. Nauka, Moscow 1971.
[12] A. A. MARADUDIN, Solid State Phys. 18, 273 (1966); 19, 1 (1966).