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Cyclotron resonance of a two-dimensional Wigner crystal

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Abstract. The scattering of two-dimensional electrons forming a Wigner crystal by vibrations of a medium (semiconductor or liquid helium) is considered. A method is developed that gives explicit expressions (including essential corrections to the Born approximation) for the cyclotron resonance (CR) peak shape. The halfwidth and shift of the peak are analysed in detail for the cases of relatively low and high electron densities (or, respectively, strong and weak magnetic fields). The results show that at low densities CR may be used to detect the short-range order in the electron system, while at high densities the transverse sound velocity may be determined for the Wigner crystal. The theory describes quantitatively the specific dependences of the CR parameters on the electron density observed experimentally for electrons above the liquid helium surface.

1. Introduction

In a number of systems (e.g., in semiconductor surface inversion layers or at the surface of liquid helium) the motion of carriers is quantised in one direction, and at sufficiently low densities and temperature the carriers behave as two-dimensional (2D) ones. Since the 2D carrier density N may be varied easily, such systems are suitable for the investigation of many-electron effects. One of these effects is Wigner crystallisation of the carriers. In the 2D case it was considered theoretically by Crandall and Williams (1971) and by Chaplik (1972) and observed experimentally by Grimes and Adams (1979) for 2D electrons above the helium surface.

The electron–electron interaction influences not only the ground state, but also the scattering of electrons by medium vibrations (MV) (phonons in semiconductors or capillary waves on the helium surface) and by impurities. The influence on scattering is extremely strong when the quantising magnetic field $H_{\perp}$ is applied normally to the electron (or hole) layer. In this case in the single-electron approximation the electron energy spectrum presents a set of Landau levels. Due to the discreteness of the spectrum elastic scattering by impurities and quasi-elastic scattering by MV are forbidden in such an approximation. The Coulomb interaction of 2D electrons makes their spectrum continuous and thus allows them to scatter elastically by outer scatterers. Such scattering was considered earlier (Dykman and Khazan 1979) for the low-density (or large-$H_{\perp}$) range where the electron motion might be described quasiclassically using the concept of the drift of the cyclotron orbit centres in the electric field caused by electron–electron interactions.

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In what follows the scattering by MV in transverse magnetic fields is analysed for a wide temperature and density range supposing that 2D electrons form a Wigner solid (WS). Here the problem of electron relaxation reduces to the problem of relaxation of the WS eigenmodes, which is specific. This follows from the explicit form of the Hamiltonian of coupling to MV:

$$H_i = \sum_q V_q c_q \rho_q \qquad c_q = b_q + b_{-q}^+ \qquad \rho_q = \sum_n \exp(i\boldsymbol{q}\boldsymbol{r}_n). \tag{1.1}$$

Here c_q denotes the MV field operator (MV quantum numbers other than the 2D wavevector $\boldsymbol{q}$ are omitted for brevity); $\boldsymbol{r}_n$ is the position vector of the nth electron. For electrons forming a WS $\boldsymbol{r}_n$ are expressed in terms of the WS phonon creation and annihilation operators $a_{k\alpha}^+$, $a_{k\alpha}$:

$$\boldsymbol{r}_n = \boldsymbol{R}_n + \boldsymbol{u}_n \qquad \boldsymbol{u}_n = \sum_{k\alpha} \boldsymbol{u}_{k\alpha} \exp(i\boldsymbol{k}\boldsymbol{R}_n) \qquad \boldsymbol{u}_{k\alpha} = \boldsymbol{A}_{k\alpha} a_{k\alpha} + \boldsymbol{A}_{-k\alpha}^* a_{-k\alpha}^+ \tag{1.2}$$

(α and $\boldsymbol{k}$ label the phonon branches and wavevectors respectively).

The distinctive feature of the coupling to MV obvious from equations (1.1) and (1.2) is its non-linearity in the WS normal coordinates $\boldsymbol{u}_{k\alpha}$. This non-linearity is strong, since the mean-square electron displacement $\langle u_n^2 \rangle$ diverges for a 2D solid at finite temperature. Therefore many WS phonons participate in each collision event and the non-linearity of H_i in $\boldsymbol{u}_{k\alpha}$ must be taken into account to all orders of perturbation theory. This removes divergences in the expressions for the relaxation parameters (for the Born approximation to H_i this was shown previously; see Dykman (1980a) and Eguiluz *et al* (1981)).

Another distinctive feature of scattering of the WS phonons by MV is the non-conservation of quasi-momentum: the MV momentum may be transferred not to WS modes but to the electron lattice as a whole (there is an analogy with the Mössbauer effect here). The Umklapp processes are very essential, because for real systems the characteristic distance between electrons, $N^{-1/2}$, exceeds substantially the atomic spacing in the medium and therefore the MV wavenumber q may exceed the maximum WS wavenumber $k_{\max} \sim N^{1/2}$. In particular, those MV with $q \gg N^{1/2}$ (and thus with relatively large density of states) are shown below to be responsible for the cyclotron resonance broadening and shift in quantising magnetic fields.

In the present paper the cyclotron resonance (CR) is analysed using the double-time Green functions. This method allows us to calculate the CR peak shape parameters taking essential corrections to the Born approximation into account. The general expression for the CR peak shape obtained in § 2 is used in §§ 3 and 4 to analyse the halfwidth and shift of the peak at relatively high and low electron densities respectively. In § 5 the results are illustrated for a simple coupling model. In § 6 they are applied to electrons on the helium surface and compared with experiment. Section 7 contains a discussion of our results.

2. General expression for the CR peak shape

2.1. Resonant conductivity of 2D electrons

The Hamiltonian for 2D electrons forming a Wigner crystal and interacting with vibrations of a medium is of the form

$$\mathcal{H} = H_0 + H_v + H_i \qquad H_0 = \sum_{k\alpha} \omega_{k\alpha} a_{k\alpha}^+ a_{k\alpha}$$
$$H_v = \sum_q \omega_q b_q^+ b_q \qquad (\hbar = 1) \tag{2.1}$$

where H_i is given by equation (1.1); the medium is supposed to be isotropic. In the transverse magnetic field the WS phonon dispersion law at small k is given by

$$\omega_{k1} \simeq \omega_c + \frac{\omega_p^2}{2\omega_c}\frac{k}{N^{1/2}} \qquad (k \ll N^{1/2})$$

$$\omega_{k,-1} \simeq c_t \omega_p k^{3/2}/\omega_c N^{1/4} \qquad (\omega_{k,-1} \ll \omega_c, \omega_c^2/\omega_p)$$

$$\omega_{k,-1} \simeq c_t k \qquad (\omega_c^2/\omega_p \ll \omega_{k,-1} \ll \omega_p) \tag{2.2}$$

$$\omega_p = (2\pi e^2 N^{3/2}/m)^{1/2} \qquad \omega_c = eH_-/mc$$

(see Chaplik 1972). Here c_t and ω_p ($c_t \sim \omega_p/N^{1/2}$) denote the transverse sound velocity and characteristic Debye frequency of a WS at $\omega_c = 0$. The branch $\alpha = 1$ at $k \ll N^{1/2}$ corresponds to magnetoplasma waves in a 2D plasma. The existence of the branch $\alpha = -1$ is connected with the electron crystallisation.

The electron conductivity $\sigma(\boldsymbol{k}, \omega)$ is determined (see Zubarev 1960) by the double-time Green function for the momentum operator $\boldsymbol{p}(\boldsymbol{k})$:

$$\sigma_{\kappa\kappa'}(\boldsymbol{k}, \omega) = i(e^2N^2S/m^2\omega)\,\langle\langle p_\kappa(\boldsymbol{k}); p_{\kappa'}(-\boldsymbol{k})\rangle\rangle_{\omega - i0} \qquad \kappa, \kappa' = x, y$$

$$\boldsymbol{p}(\boldsymbol{k}) = -im\sum_\alpha \omega_{k\alpha}(\boldsymbol{A}_{k\alpha}a_{k\alpha} - \boldsymbol{A}^*_{-k\alpha}a^-_{-k\alpha}) \tag{2.3}$$

where

$$\langle\langle X; Y\rangle\rangle_\omega = -i\int_0^\infty dt \exp(i\omega t)\,\langle[X(t), Y(0)]\rangle$$

(S is the area of the system, $\langle \ldots \rangle$ denotes statistical averaging).

Radiation absorption in general and CR in particular are described by Re $\sigma_{xx}(\boldsymbol{k}, \omega)$. To consider CR we must find Re $\sigma_{xx}(\boldsymbol{k}, \omega)$ in the limit $k \to 0$. In this limit the coefficient $\boldsymbol{A}_{k1}$ in equation (2.3) is given by

$$\boldsymbol{A}_{k1} = il(2NS)^{-1/2}\left(\frac{\boldsymbol{k}}{k} + i\frac{\boldsymbol{k}\times\boldsymbol{H}_-}{kH_-}\right) \qquad k \to 0$$

$$l = (m\omega_c)^{-1/2} = (c/eH_-)^{1/2} \tag{2.4}$$

while $\omega_{k,-1}A_{k,-1} \propto k^{5/4}$ (we have used here the general expressions for $\boldsymbol{A}_{k\alpha}$ obtained by Ulinich and Usov (1979)). Note that $(NS)^{1/2}A_{k1}$ at $k \to 0$ equals the matrix element of the coordinates of an isolated 2D electron on the wavefunctions of the ground and first excited Landau levels (this may be easily understood because the Coulomb interaction causing Wigner crystallisation should not influence the elementary excitations with $k = 0$). Since $\omega_{k,-1}A_{k,-1}$ vanishes at $k \to 0$, only the phonons of the branch $\alpha = 1$ contribute to the momentum $\boldsymbol{p}(\boldsymbol{k})$ and hence to the conductivity $\sigma(\boldsymbol{k}, \omega)$ in the long-wavelength limit.

If the CR peak halfwidth Γ is small compared with ω_c, in the range of the peak ($\omega \sim \omega_c$), according to equations (2.3) and (2.4), we have

$$\text{Re}\,\sigma_{xx}(\omega) \simeq (Ne^2/2m)\,Q(\omega) \qquad Q(\omega) = -\lim_{k\to 0}\text{Im}\,Q_k(\omega + i0)$$

$$Q_k(\omega) = \langle\langle a_{k1}; a^+_{k1}\rangle\rangle_\omega \qquad \omega \sim \omega_c. \tag{2.5}$$

To obtain (2.5) we have replaced ω in the denominator of expression (2.3) for $\sigma_{xx}(\boldsymbol{k}, \omega)$ by ω_c and have neglected the Green functions $\langle\langle a_{k1}; a_{-k1}\rangle\rangle_\omega$, $\langle\langle a^+_{k1}; a_{k1}\rangle\rangle_\omega$ and $\langle\langle a^+_{-k1}; a^+_{k1}\rangle\rangle_\omega$.

All these functions are of order ω_c^{-1} at $\omega \sim \omega_c$, while to zeroth order in H_i the function $Q_k(\omega)$ retained in equation (2.5) has a pole at $\omega = \omega_{k1} \simeq \omega_c(k \to 0)$. The coupling to MV transforms this pole into the peak of $-\mathrm{Im}\, Q_k(\omega + \mathrm{i}0)$. Within the peak ($|\omega - \omega_c| \lesssim \Gamma$) $Q(\omega) \sim \Gamma^{-1}$ and thus all corrections to $\mathrm{Re}\, \sigma_{xx}(\boldsymbol{k}, \omega)$ omitted in equation (2.5) are of the order of Γ/ω_c, $|\omega - \omega_c|/\omega_c \ll 1$. The function $Q(\omega)$ gives the CR peak shape itself.

2.2. *Evaluation of the peak shape*

2.2.1. Equation of motion for the Green function $Q_k(\omega)$. To calculate $Q_k(\omega)$ it is convenient to single out the terms containing $\boldsymbol{u}_{k1}$, $\boldsymbol{u}_{-k1}$ in the interaction Hamiltonian (1.1):

$$H_i = H_i^{(1)} + H_i^{(2)} \qquad H_i^{(\nu)} = \sum_q V_q c_q \rho_q^{(\nu)} \qquad \nu = 1, 2$$

$$\rho_q^{(1)} = \mathrm{i}[(\boldsymbol{q}\boldsymbol{u}_{k1})\rho_q'(\boldsymbol{k}) + (\boldsymbol{q}\boldsymbol{u}_{-k1})\rho_q'(-\boldsymbol{k})] \qquad \rho_q^{(2)} = [1 - (\boldsymbol{q}\boldsymbol{u}_{k1})\,(\boldsymbol{q}\boldsymbol{u}_{-k1})]\rho_q'$$

$$\rho_q'(\boldsymbol{k}) = \sum_n \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}_n' + \mathrm{i}\boldsymbol{k}\boldsymbol{R}_n) \qquad \rho_q' \equiv \rho_q'(0)$$

$$\boldsymbol{r}_n' = \boldsymbol{r}_n - (\boldsymbol{u}_{k1} \exp(\mathrm{i}\boldsymbol{k}\boldsymbol{R}_n) + \mathrm{HC}). \tag{2.6}$$

We stress that equation (2.6) is not an expansion of H_i in electron displacements (such an expansion diverges); H_i is expanded only in one Fourier component of the displacement. Since $A_{k1} \propto S^{-1/2}$ (cf equation (2.4)), only the terms up to second order in $u_{\pm k1}$ are taken into account in equation (2.6); the contribution to $Q_k(\omega)$ from the higher-order terms in $u_{\pm k1}$ is shown below to vanish in the statistical limit $S \to \infty$.

The coupling Hamiltonian $H_i^{(1)}$ has the usual form for interacting vibrations and describes the decay of the WS phonon $\{\boldsymbol{k}1\}$ into, generally speaking, several other WS phonons and an MV quantum. The term $H_i^{(2)}$ has a distinctive structure: it contains the addend $\propto u_{k1}u_{-k1}$. Although the latter is proportional to S^{-1} it must be taken into account, because in $\rho_q^{(2)}$ there is the addend (ρ_q') which does not contain S in the denominator. The terms $u_{\pm k1}^2$ in $\rho_q^{(2)}$ are omitted since their convolution with the large term ρ_q' vanishes (this is obvious: the coefficient at $u_{\pm k1}^2$ bears the momentum $\pm 2\boldsymbol{k}$ while ρ_q' is independent of $\boldsymbol{k}$).

Two terms in the Hamiltonian (2.6) correspond to two Green functions on the RHS of the equation of motion for $Q_k(\omega)$ ($k \ll N^{1/2}$):

$$(\omega - \omega_c)\, Q_k(\omega) = 1 + \langle\langle \hat{A}; a_{k1}^+ \rangle\rangle_\omega + \langle\langle \hat{B}a_{k1}; a_{k1}^+ \rangle\rangle_\omega$$

$$\hat{A} = i \sum_q (\boldsymbol{q}\boldsymbol{A}_{k1}^*)\, V_q c_q \rho_q'(-\boldsymbol{k}) \qquad \hat{B} = -\frac{1}{2NS} \sum_q l^2 q^2 V_q c_q \rho_q'. \tag{2.7}$$

This equation is obtained by differentiating $[a_{k1}(t), a_{k1}^+(0)]$ with respect to t, averaging, and Fourier transformation over t; we have replaced $|\boldsymbol{q}\boldsymbol{A}_{k1}|^2$ in $\hat{B}$ by $l^2q^2/2NS$ using equation (2.4). The Green function $\Sigma_q(\boldsymbol{q}\boldsymbol{A}_{k1}^*)^2 V_q \langle\langle c_q \rho_q' a_{-k1}^+; a_{k1} \rangle\rangle_\omega$ omitted on the RHS of equation (2.7) vanishes for an isotropic medium in the limit $k \to 0$, $S \to \infty$ (cf the calculations carried out below).

2.2.2. Calculation of $\langle\langle \hat{B}a_{k1}; a_{k1}^+ \rangle\rangle_\omega$. The Green function $\langle\langle \hat{B}a_{k1}; a_{k1}^+ \rangle\rangle_\omega$ may be expressed in terms of $Q_k(\omega)$ strictly for arbitrary coupling. This is a consequence of the specific structure of $\hat{B}$: it presents the sum over electrons of the single-electron operators $(2NS)^{-1}\Sigma_q l^2 q^2 V_q c_q \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}_n')$. Thus the operator $S\hat{B}$ is additive.

It follows from equation (1.2) that

$$\sum_{\alpha} \boldsymbol{u}_{k\alpha} = (NS)^{-1} \sum_{n} \exp(-\mathrm{i}\boldsymbol{k}\boldsymbol{R}_n)\boldsymbol{u}_n$$

and therefore $SA_{k\alpha}a_{k\alpha}$ and $SA^{*}_{k\alpha}a^{+}_{k\alpha}$ are also additive operators. Then to calculate $\langle\langle \hat{B}a_{k1}; a^{+}_{k1}\rangle\rangle_{\omega}$ we must average the product of three additive operators $S\hat{B}(t)$, $S^{1/2}a_{k1}(t)$ and $S^{1/2}a^{+}_{k1}(0)$.

The average of the product of arbitrary additive operators X_i may be easily expressed as a power series in S^{-1}. The beginning of this series is of the form

$$\left\langle \prod_i X_i \right\rangle = \prod_i \langle X_i\rangle + \sum_i \sum_{\substack{j>k \\ (j,k\neq i)}} \langle X_i\rangle \langle X_jX_k\rangle_{\mathrm{c.a.}} + \ldots$$

$$\langle X_jX_k\rangle_{\mathrm{c.a.}} = \langle X_jX_k\rangle - \langle X_j\rangle \langle X_k\rangle \tag{2.8}$$

(this expression is well known; it may be proved by dividing the system into a large number of macroscopic (and thus practically uncoupled) subsystems, see, e.g., Landau and Lifshitz (1980)).

The terms omitted in (2.8) are also of higher order in S^{-1} for the case when the operators X_i are taken at various times. Then, taking into account that $\langle a_{k1}\rangle = \langle a^{+}_{k1}\rangle = 0$ (there are no electron fluxes in the system), one obtains in the limit $S \to \infty$

$$\langle \hat{B}(t)a_{k1}(t)\, a^{+}_{k1}(0)\rangle = \langle \hat{B}\rangle \langle a_{k1}(t)\, a^{+}_{k1}(0)\rangle$$

and therefore

$$\langle\langle \hat{B}a_{k1}; a^{+}_{k1}\rangle\rangle_{\omega} = P^{(2)}Q_k(\omega) \qquad P^{(2)} = \langle \hat{B}\rangle = -\frac{1}{2NS}\sum_{q} l^2q^2V_q\langle c_q\rho_q\rangle \tag{2.9}$$

(with accuracy to S^{-1} the operator ρ'_q is replaced by ρ_q here).

Thus the function $\langle\langle \hat{B}a_{k1}; a^{+}_{k1}\rangle\rangle_{\omega}$ is directly proportional to $Q_k(\omega)$. The coefficient $P^{(2)}$ is an analogue of the average coupling energy per electron; it is independent of S in the limit $S \to \infty$.

Equation (2.8) allows us to show that the contribution to $Q_k(\omega)$ of terms in H_i of third and higher order in $\boldsymbol{u}_{k1}$ is negligible. This is a consequence of Wick's theorem. It should be noted, however, that in contrast to the usual problems of interacting phonons the term $\propto |u_{k1}|^2$ in the expansion (2.6) is essential for the WS relaxation.

2.2.3. General expressions for the CR peak parameters. When the condition $\omega_c \gg \Gamma$ is fulfilled, it is sufficient to calculate $\langle\langle \hat{A}; a^{+}_{k1}\rangle\rangle_{\omega}$ in the Born approximation. Then

$$Q_k(\omega) = (\omega - \omega_c - \Pi^{(1)}(\omega) - P^{(2)})^{-1} \qquad \omega \sim \omega_c \tag{2.10}$$

where, in the limit $k \to 0$,

$$\Pi^{(1)}(\omega) = -\tfrac{1}{2}\mathrm{i}\sum_{q} l^2q^2|V_q|^2 \int_0^{\infty} \mathrm{d}t \exp(\mathrm{i}\omega t)\Pi_q(t) \qquad \Pi_q(t) = \zeta_q(t)\varphi(\omega_q, t) - \mathrm{cc}$$

$$\zeta_q(t) = \langle \rho_q(t)\rho_{-q}(0)\rangle_i/NS \qquad \varphi(\omega, t) = (\bar{n}(\omega) + 1)\exp(-\mathrm{i}\omega t) + \bar{n}(\omega)\exp(\mathrm{i}\omega t)$$

$$\langle X_1(t)\, X_2(0)\rangle_i \equiv Z_0^{-1}\,\mathrm{Tr}\left[\exp\left(-\frac{H_0}{T} + \mathrm{i}H_0t\right)X_1 \exp(-\mathrm{i}H_0t)X_2\right]$$

$$Z_0 = \mathrm{Tr}\exp(-H_0/T) \qquad \bar{n}(\omega) = (\exp(\omega/T) - 1)^{-1}. \tag{2.11}$$

The polarisation operator $\Pi^{(1)}(\omega)$ is caused by the coupling $H_i^{(1)}$. The kernel $\Pi_q(t)$ contains the product of the time correlation function of the MV coordinates $\varphi(\omega_q, t)$ with that of the electron density $\langle\rho_q(t)\rho_{-q}(0)\rangle_i$, both calculated neglecting the electron–MV interaction (strictly speaking, $\Pi_q(t)$ contains the correlator of $\rho_q'(\boldsymbol{k})$, $\rho_{-q}'(-\boldsymbol{k})$ instead of the correlator of ρ_q, ρ_{-q}, but in the limit $k \to 0$ they coincide). The approximate calculation of the contribution of $H_i^{(1)}$ to $Q_q(\omega)$ is justified if $\Pi^{(1)}(\omega)$ is small and smooth:

$$|\Pi^{(1)}(\omega)| \ll \omega_c \qquad |\partial\Pi^{(1)}(\omega)/\partial\omega| \ll 1 \qquad (\omega \sim \omega_c). \tag{2.12}$$

It follows from equations (2.5), (2.10) and (2.12) that at $\omega \sim \omega_c$

$$Q(\omega) = \Gamma[(\omega - \omega_c - P)^2 + \Gamma^2]^{-1}$$

$$\Gamma = -\operatorname{Im}\Pi^{(1)}(\omega_c + i0) \qquad P = P^{(1)} + P^{(2)}$$

$$P^{(1)} = \operatorname{Re}\Pi^{(1)}(\omega_c + i0). \tag{2.13}$$

Thus the CR peak shape is Lorentzian. Its broadening Γ is due to the decay coupling $H_i^{(1)}$ while both $H_i^{(1)}$ and $H_i^{(2)}$ contribute to the shift P.

2.3. Calculation of $P^{(2)}$

The expression (2.9) for $P^{(2)}$ simplifies in the actual case of low electron densities when the medium may be considered to be a thermostat for the electrons and the damping of medium vibrations due to their interaction with the WS may be neglected. In this case the following decoupling holds:

$$\langle H_q\rangle \simeq -\int_0^\lambda d\tau\langle H_{-q}(-i\tau)H_q(0)\rangle \simeq -|V_q|^2\int_0^\lambda d\tau\,\varphi(\omega_q, -i\tau)\,\langle\rho_{-q}(-i\tau)\rho_q(0)\rangle$$

$$H_q = V_q c_q \rho_q \qquad \lambda = T^{-1}$$

(to calculate $\langle H_q\rangle$ it is convenient to single out $H_{\pm q}$ with a given $\boldsymbol{q}$ in H_i and to expand $\exp(-\lambda\mathcal{H})$ in $H_q + H_{-q}$; it is sufficient to take only the linear term in this expansion into account and to neglect $\langle H_q(-i\tau)H_q(0)\rangle$ when the MV renormalisation is neglected). In fact, the expansion parameter used here is the ratio of the number of 2D electrons to that of the molecules participating in the medium vibrations coupled to the electrons. Substitution of the expression for $\langle H_q\rangle$ given above into equation (2.9) yields

$$P^{(2)} = \frac{1}{2NS}\sum_q l^2q^2|V_q|^2\int_0^\lambda d\tau\,\varphi(\omega_q, -i\tau)\,\langle\rho_q(-i\tau)\rho_{-q}(0)\rangle \qquad (\lambda = T^{-1}). \tag{2.14}$$

The explicit expression (2.14) for $P^{(2)}$, in contrast to equation (2.11) for $\Pi^{(1)}(\omega)$, contains the electron density time correlation function calculated with the complete electron–MV Hamiltonian (2.1). In the Born approximation this correlator should be calculated neglecting coupling:

$$P_B^{(2)} = \frac{1}{2NS}\sum_q l^2q^2|V_q|^2\int_0^\lambda d\tau\,\varphi(\omega_q, -i\tau)\,\langle\rho_q(-i\tau)\,\rho_{-q}(0)\rangle_i = -\operatorname{Re}\Pi^{(1)}(+i0) \tag{2.15}$$

(the second equality follows from (2.11) and the Kubo identity).

It is shown below that even for $\Gamma, |P| \ll \omega_c$ the difference between $P^{(2)}$ and $P_B^{(2)}$ may be essential, i.e. the Born approximation may be insufficient for the self-energy.

2.4. 'Single-site' approximation for the self-energy

The electron density time correlation function for a WS isolated from the medium according to equations (1.1), (1.2), (2.1) and (2.11) is given by the expression

$$\zeta_q(t) = \sum_n \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{R}_n + W_q(\boldsymbol{R}_n, t))$$

$$W_q(\boldsymbol{R}_n, t) = -\sum_{k\alpha} |\boldsymbol{q}\boldsymbol{A}_{k\alpha}|^2 [(\bar{n}_{k\alpha} + 1)(1 - \exp(-\mathrm{i}\omega_{k\alpha}t - \mathrm{i}\boldsymbol{k}\boldsymbol{R}_n)) + \bar{n}_{k\alpha}(1 - \exp(\mathrm{i}\omega_{k\alpha}t + \mathrm{i}\boldsymbol{k}\boldsymbol{R}_n))] \qquad \bar{n}_{k\alpha} \equiv \bar{n}(\omega_{k\alpha}). \tag{2.16}$$

It is obvious from equations (2.11), (2.15) and (2.16) that the evaluation of $\Pi^{(1)}(\omega)$, $P_B^{(2)}$ includes the summation over electron sites $\boldsymbol{R}_n$ and MV momenta $\boldsymbol{q}$. In the present paper we suppose that $q^2|V_q|^2/\omega_q$ does not increase with decreasing q, and $\omega_c \gg (\omega_q)_{q=G_0}$ where $\boldsymbol{G}_0$ is the minimum WS reciprocal lattice vector. In quantising magnetic fields these conditions are satisfied very well both for 2D electrons in non-polar semiconductors and at the helium surface (Dykman 1978, 1980b). Then it is convenient to sum first over $\boldsymbol{q}$ in (2.11) and (2.15).

The addend in $\zeta_q(t)$ corresponding to $R_n = 0$ is of the form

$$\bar{\zeta}_q(t) = \exp(q^2 W(t))$$

$$W(t) = \tfrac{1}{2}(\langle \boldsymbol{u}_n(t)\boldsymbol{u}_n(0)\rangle_i - \langle u_n^2\rangle_i) = \tfrac{1}{2}\sum_{k\alpha} |A_{k\alpha}|^2(\varphi(\omega_{k\alpha}, t) - \varphi(\omega_{k\alpha}, 0)); \tag{2.17}$$

it falls off exponentially with q^2. The main contribution to $\Pi^{(1)}(\omega)$ results from $\bar{\zeta}_q(t)$ with $q \sim q_s = u_s^{-1}$ where u_s is the electron displacement over the time t_s equal to the duration of the collision with MV: $u_s = (-W(t_s))^{1/2}$ (obviously $q_s \gg N^{1/2}$). The contribution of $\bar{\zeta}_q(t)$ with $q \ll N^{1/2}$ is small, since the density of states for the corresponding MV is small.

The contribution of the terms with $R_n \neq 0$ in $\zeta_q(t)$ to $\Pi^{(1)}(\omega)$ for $\omega \sim \omega_c$ is shown in the Appendix to be exponentially small for actual systems. This estimate (equation (A.5)) holds also for $\mathrm{Re}\,\Pi^{(1)}(+\mathrm{i}0)$, i.e. for $P_B^{(2)}$. Thus the kernel $\Pi_q(t)$ in equation (2.11) may be replaced by

$$\bar{\Pi}_q(t) = \bar{\zeta}_q(t)\,\varphi(\omega_q, t) - \text{cc}. \tag{2.18}$$

This means the neglect of interference of short-wavelength ($q \gg N^{1/2}$) vibrations of a medium scattered by different electrons. It should be noted that the factor $\exp(q^2W(t))$ in equation (2.17) is analogous to a similar factor in the theory of the Mössbauer effect. In both cases the momentum of the scattered particle (MV or γ quantum) is transferred to the lattice as a whole.

Below we shall analyse the most real and interesting case of strong magnetic fields

$$\omega_c \gg \omega_s \qquad (\omega_s \equiv \omega_{q_s}), \qquad \exp(\omega_c/T) \gg 1 \tag{2.19}$$

when the scattering of the phonon $\{\boldsymbol{k}1\}$ ($k \to 0$) by MV is quasi-elastic (the energy $\hbar\omega_c$ of the phonon passes into the energy of other WS phonons).

3. CR peak halfwidth and shift at low densities

3.1. Electron displacement correlator

In the range of strong magnetic fields or relatively low densities where $\omega_p \ll \omega_c$ ($\omega_p = (2\pi e^2 N^{3/2}/m)^{1/2}$) the widths of both WS phonon branches $\alpha = \pm 1$ (and hence the maximum

frequency of the soft branch $\alpha = -1$) are of order ω_p^2/ω_c. The expressions for Γ and P simplify if the vibrations of the branch $\alpha = -1$ are classical:

$$\omega_p \ll \omega_c \qquad \omega_p^2/\omega_c \ll T. \tag{3.1}$$

For $\omega_p^2/\omega_c \ll T$ the main contribution to the integral (2.11) over t comes from the range $t \ll \omega_c/\omega_p^2$ (see below). In this range the displacement correlator $W(t)$ may be expanded in $\omega_{k,-1}t$ and $(\omega_{k1} - \omega_c)t$:

$$W(t) \simeq -\tfrac{1}{2}l^2(1 - \exp(-i\omega_c t)) - \frac{\pi l^2}{4\tau_e^2}\left(t^2 + \frac{it}{T}(1 + \exp(-i\omega_c t))\right) \qquad t \ll \omega_c/\omega_p^2$$

$$\tau_e = \left(\frac{\pi\omega_c}{T}\right)^{1/2}\left(\frac{1}{NS}\sum_{\boldsymbol{k}}(\omega_l^2(\boldsymbol{k}) + \omega_t^2(\boldsymbol{k}))\right)^{-1/2} \tag{3.2}$$

($\omega_{l,t}(\boldsymbol{k})$ denote the frequencies of WS longitudinal and transverse phonons at $H_\perp = 0$). We have utilised here the inequalities $\omega_{k,-1} \ll T$, $\exp(\omega_{k1}/T) \gg 1$ and the relationships

$$|A_{k1}|^2 \simeq l^2/NS \qquad \frac{l^2}{NS}\sum_{\boldsymbol{k}}(\omega_{k1} - \omega_c) \simeq \sum_{\boldsymbol{k}}\omega_{k,-1}|A_{k,-1}|^2 \simeq \frac{l^2}{2NS\omega_c}\sum_{\boldsymbol{k}}(\omega_l^2(\boldsymbol{k}) + \omega_t^2(\boldsymbol{k}))$$

following at $\omega_p \ll \omega_c$ from the expressions for $\omega_{k\alpha}$ and $|A_{k\alpha}|^2$ given by Ulinich and Usov (1979).

According to equations (2.11), (2.17) and (3.2) the momentum q_s of the MV essential for scattering equals l^{-1}, while the duration of the collision $t_s = \tau_e$. The well known sum rule for the squares of phonon frequencies allows us to recast τ_e as

$$\tau_e = (l/c)H_\perp\langle E^{-1}\rangle \qquad \langle E^{-1}\rangle = (\pi e^2/2T\Lambda_0)^{1/2}$$

$$\Lambda_0 = \tfrac{1}{2}e^2\nabla_n^2\sum_{m(m\neq n)}|\boldsymbol{R}_n - \boldsymbol{R}_m|^{-1} = \delta e^2 N^{3/2} \qquad (\nabla_n \equiv \partial/\partial\boldsymbol{R}_n). \tag{3.3}$$

Obviously only a few coordination spheres contribute to Λ_0 and hence to τ_e; the direct calculation gives $\delta \simeq 4.45$ for the hexagonal lattice and $\delta \simeq 4.52$ for the square one.

On a vibrating electron in the WS acts the electric field of other electrons. For $\omega_c \gg \omega_p$ it may be divided into slow and fast (in the scale ω_c^{-1}) components. The slow component, $\boldsymbol{E}$, at $\omega_p^2/\omega_c \ll T < \omega_c$ may be shown to have a Gaussian distribution analogous to that of the restoring force in the WS for $T \gg \theta_D$ and $H_\perp = 0$ (θ_D is the Debye temperature). In particular $\langle E^{-1}\rangle$ is given by equation (3.3). Then τ_e should be interpreted as the time needed for a 2D electron to drift in the crossing fields $\boldsymbol{E}, \boldsymbol{H}_\perp$ over the distance l equal to the characteristic MV wavelength. Obviously $\tau_e \sim (T\omega_p^2/\omega_c)^{-1/2} \ll \omega_c/\omega_p^2$. Thus in the actual range of t ($t \lesssim t_s = \tau_e$) where $q_s^2|W(t)| \lesssim 1$ the inequality $t \ll \omega_c/\omega_p^2$ used in equation (3.2) is fulfilled.

3.2. Explicit expressions for the CR peak parameters

To calculate the CR broadening and shift the expression (3.2) for $W(t)$ should be substituted into equation (2.17). Then to integrate over t in equations (2.11) and (2.15) it is convenient to expand $\xi_q(t)$ in terms of $\exp(-i\omega_c t)$. The main contribution to Γ results from the linear term in this expansion (the remainder give a correction of order $\exp(-\omega_c^2\tau_e^2) \ll 1$) and in the actual case $\omega_s\tau_e \ll 1$ takes the form

$$\Gamma = \tfrac{1}{2}T\tau_e\sum_{\boldsymbol{q}}(lq)^3|V_q|^2\omega_q^{-1}\exp(-\tfrac{1}{2}l^2q^2) \qquad T \gg \tau_e^{-1} \gg \omega_s, \omega_p^2/\omega_c. \tag{3.4}$$

The CR peak halfwidth (3.4) is due to decay of the long-wavelength phonon $\{k1\}$ ($k \to 0$) into a short-wavelength phonon of the same branch $\alpha = 1$. The decay is induced by MV and is accompanied by the creation and annihilation of a great number ($\sim \omega_c/\omega_p^2\tau_e \gg 1$) of phonons $\alpha = -1$.

It is obvious from equations (3.2), (2.11), (2.13) and (2.15) that the CR peak shift P is determined by the terms proportional to $\omega_s\tau_e \ll 1$ or to $(\hbar\omega_p^2/\omega_c T)^{1/2} \ll 1$ (i.e. by the quantum corrections omitted in equation (3.4)). Expanding $\exp(q^2W(t))$ in terms of $t/T\tau_e^2$ (up to the linear term) and in $\exp(-i\omega_c t)$ we obtain

$$P = P_0 + P_e + P_T$$

$$P_0 = \sum_q F_q\lambda_q(1-\lambda_q) \qquad P_e = \frac{2}{\pi}\sum_q F_q(\omega_q\tau_e)^2(1-\tfrac{1}{2}\lambda_q)$$

$$P_T = \frac{2T}{\omega_c}\sum_q F_q\lambda_q^2\left(\tfrac{3}{2} - 2\sum_{m=1}^{\infty}\lambda_q^m[m(m+1)(m+2)!]^{-1}\right) \qquad (T \gg \tau_e^{-1} \gg \omega_s, \omega_p^2/\omega_c)$$

$$F_q = |V_q|^2\omega_q^{-1}\exp(-\lambda_q) \qquad \lambda_q = \tfrac{1}{2}l^2q^2 \qquad (3.5)$$

(we have put $\varphi(\omega_q, t) = (2T/\omega_q) - i\omega_q t$ in equation (2.11)). The expression for P_0 is similar to that for the single-electron CR shift due to virtual processes of MV creation and annihilation without transitions between Landau levels (cf Dykman 1978). The difference in the expressions and the addend P_e are caused by many-electron effects. The term P_T is due to virtual processes where phonons with $\alpha = 1$ are created or annihilated together with an MV quantum. This term coincides with the corresponding term in the single-electron approximation because $\omega_{k1} \simeq \omega_c \gg \tau_e^{-1}$.

For $\omega_p^2/\omega_c \ll T$ and $\omega_s\tau_e \ll 1$ the CR peak halfwidth Γ substantially exceeds the shift P. Equations (3.4) and (3.5) (i.e. the Born approximations for Γ and P) are valid at arbitrary Γ/ω_s, but for $\Gamma\tau_e \ll 1$. For $\Gamma\tau_e \gg 1$ the interaction between electrons may be neglected when the CR is considered and the single-electron strong-coupling theory (Dykman 1978) should be used.

4. CR halfwidth and shift at high densities

4.1. Electron displacement correlator

At high densities or relatively weak magnetic fields where

$$\omega_p \gg \omega_c \qquad (4.1)$$

the characteristic width of both WS vibrational branches is equal to ω_p. Hence the spectrum of $W(t)$ is continuous and smooth up to frequencies of order ω_p. Therefore the lowest frequency singled out under the integral over t in equation (2.11) is $|\omega \pm \omega_q|$ ($|\omega| \ll \omega_p$), and the characteristic duration of scattering t_s is equal to ω_c^{-1} for $\omega \sim \omega_c \gg \omega_s$.

In the range $t \lesssim \omega_c^{-1}$ the expression (2.17) for $W(t)$ takes the form

$$W(t) \simeq -\eta + w(t) \qquad t \lesssim \omega_c^{-1} < T^{-1} \qquad \eta = \tfrac{1}{2}\sum_{k\alpha}|A_{k\alpha}|^2$$

$$w(t) = \tfrac{1}{2}\sum_{k\alpha}|A_{k\alpha}|^2\exp(-i\omega_{k\alpha}t) \qquad \eta \equiv w(0). \qquad (4.2)$$

The relationship

$$|A_{k,-1}|^2/\omega_{k,-1} \simeq (2mNSc_t^2k^2)^{-1} \qquad (k \ll N^{1/2}) \tag{4.3}$$

obtained by Ulinich and Usov (1979) and the explicit expression (2.2) for $\omega_{k,-1}$ allow us to show easily that the temperature-dependent correction to equation (4.2) for $W(t)$ is of order $T^3t^2/\omega_p \ll T^3/\omega_c^2\omega_p \ll 1$.

It follows from equations (4.2), (2.17) and (2.11) that the characteristic momentum of the MV essential for scattering is equal to $\eta^{-1/2}$. The parameter η is the mean-square electron displacement (divided by two) due to quantum (i.e. zero-temperature) fluctuations. It was calculated by Fukuyama (1976). For $\omega_p \gg \omega_c$

$$\eta = \mu/2m\omega_p \qquad (\mu \sim 1) \qquad q_s = \eta^{-1/2} \sim (m\omega_p)^{1/2}. \tag{4.4}$$

4.2. Broadening of the CR peak

To calculate $\Pi^{(1)}(\omega)$ at $|\omega| > T$ one should substitute $W(t)$ given by equation (4.2) into equation (2.17) and then (2.11), and expand $\exp(q^2W(t))$ in a power series in $w(t)$. Zeroth- and first-order terms yield, respectively,

$$\Pi_0^{(1)}(\omega) = \sum_q l^2q^2|V_q|^2 \exp(-\eta q^2)\, \omega_q(\omega^2 - \omega_q^2)^{-1} \tag{4.5}$$

$$\Pi_1^{(1)}(\omega) = \tfrac{1}{2}\sum_q l^2q^4|V_q|^2 \exp(-\eta q^2) \sum_{k\alpha} |A_{k\alpha}|^2\{(\bar{n}_q + 1)(\omega_{k\alpha} + \omega_q)[\omega^2 - (\omega_{k\alpha} + \omega_q)^2]^{-1}$$
$$+ \bar{n}_q(\omega_{k\alpha} - \omega_q)[\omega^2 - (\omega_{k\alpha} - \omega_q)^2]^{-1}\} \qquad |\omega| > T$$
$$\bar{n}_q \equiv \bar{n}(\omega_q). \tag{4.6}$$

The imaginary part of $\Pi_0^{(1)}(\omega_c + i0)$ yields the CR broadening Γ_0 corresponding to direct decays of the phonon $\{\boldsymbol{k}1\}$ $(k \to 0)$ into MV. For $\omega_c \gg \omega_s$ such decays are forbidden, and the main contribution to Γ comes from $\Pi_1^{(1)}(\omega)$:

$$\Gamma \simeq \Gamma_1 \qquad \Gamma_1 = -\operatorname{Im}\Pi_1^{(1)}(\omega_c + i0) = (16mc_t^2N)^{-1}\sum_q l^2q^4|V_q|^2$$
$$\times \exp(-\eta q^2)\,(2\bar{n}_q + 1) \qquad \omega_p \gg \omega_c \gg \omega_s \tag{4.7}$$

(we have used equations (2.2) and (4.3) here and neglected the corrections of order ω_c^2/ω_p^2). The broadening Γ_1 is due to decays of the phonon $\{\boldsymbol{k}1\}$ $(k \to 0)$ under consideration into one of the phonons $\{\boldsymbol{k}', -1\}$ with $k' \simeq \omega_c/c_t$. These decays are induced by coupling to a medium and are accompanied by emission or absorption of MV.

The probability of decay into several WS phonons is small compared with Γ_1. It may be calculated taking terms non-linear in $w(t)$ into account in the power series for $\exp(q^2W(t))$. The nth term yields

$$\Gamma_n = [16mc_t^2Nn!(n-1)!]^{-1}\sum_q l^2q^4|V_q|^2 \exp(-\eta q^2)\,(2\bar{n}_q + 1)\left(\frac{q^2\omega_c}{8\pi mc_t^2N}\right)^{n-1} \tag{4.8}$$

(the numerical factor of order one appearing at $n \sim \omega_p/\omega_c$ and all corrections of order ω_c^2/ω_p^2 are omitted here). Obviously $\Gamma_n \sim (\omega_c/\omega_p)^{n-1}\Gamma_1$. The numerical coefficient in Γ_n decreases rapidly with increasing n, so that the expansion $\Gamma = \Sigma_n\Gamma_n$ converges rapidly.

4.3. Shift of the CR peak

We begin with the calculation of P in the Born approximation (2.15). Then we should

find Re $\Pi^{(1)}(\omega + i0)$ at $\omega = \omega_c$ and $\omega = 0$. Since equations (4.5) and (4.6) are valid for $|\omega| \geqslant T$, to find Re $\Pi^{(1)}(+i0)$ the temperature-dependent corrections to $W(t)$ omitted in equation (4.2) should be taken into account. They are small ($\sim T/\omega_p$) in the actual range $|t| \leqslant T^{-1}$ (Re $\Pi^{(1)}(+i0)$ is determined by $W(-it)$ with $t \leqslant T^{-1}$ according to equations (2.15), (2.11) and (2.17)). This may be checked directly by expanding $\exp(q^2W(t))$ in the expression for Re $\Pi^{(1)}(+i0)$ in a series in $q^2(W(t) + \eta)$. The zeroth-order term gives an expression for Re $\Pi^{(1)}(+i0)$ which coincides with the RHS of equation (4.5) where ω is put equal to zero, while the first-order term yields

$$\operatorname{Re}\Pi_1^{(1)}(+i0) = -\tfrac{1}{2}\sum_q l^2q^4|V_q|^2\exp(-\eta q^2)\sum_{k\alpha}|A_{k\alpha}|^2[(\bar{n}_{k\alpha}+\bar{n}_q+1) \times(\omega_{k\alpha}+\omega_q)^{-1}+(\bar{n}_q-\bar{n}_{k\alpha})(\omega_{k\alpha}-\omega_q)^{-1}-2\bar{n}_{k\alpha}\omega_q^{-1}]. \tag{4.9}$$

Equations (2.13), (2.15), (4.5), (4.6) and (4.9) with allowance for the WS phonon dispersion law (2.2) and (4.3) in the case of coupling to low-frequency MV ($\omega_c \gg \omega_s$) yield

$$P \simeq P_0 + P_1 \qquad \omega_p \gg \omega_c \gg \omega_s$$

$$P_0 = \sum_q l^2q^2|V_q|^2\omega_q^{-1}\exp(-\eta q^2)$$

$$P_1 = (8\pi mc_t^2N)^{-1}\sum_q l^2q^4|V_q|^2(2\bar{n}_q+1)\exp(-\eta q^2) \times\ln(\omega_q/\tilde{\omega}) \qquad \tilde{\omega} = \max(\omega_s, T). \tag{4.10}$$

The addend P_0 here comes from $P_0^{(2)} = -$ Re $\Pi_0^{(1)}(+i0)$ (the term $P_0^{(1)} =$ Re $\Pi_0^{(1)}(\omega_c + i0)$ is omitted, $P_0^{(1)} \sim (\omega_s^2/\omega_c^2)P_0 \ll P_0$). This shift is due to virtual transitions where WS phonons do not participate explicitly. It is interesting that the expressions (4.10) and (3.5) for P_0, although similar in structure, contain different pre-exponential factors (q_s in both cases is determined from the zero-temperature mean-square electron displacement).

The shift P_1($P_1 =$Re $\Pi_1^{(1)}(\omega_c + i0) -$ Re $\Pi_1^{(1)}(+i0)$) is due to virtual processes where an MV quantum and one WS phonon are created or annihilated. The term P_1 is relatively small: $P_1/P_0 \sim (\tilde{\omega}/\omega_p)\ln(\omega_c/\tilde{\omega})$. The expression (4.10) for P_1 is valid provided $T \gg \omega_s$ (excluding a narrow temperature range where $\omega_c^2/\omega_p T \simeq 1$) or $T \ll \omega_s$ ($\tilde{\omega}$ should be replaced by ω_q then, but this yields inessential corrections for $\omega_c \gg \omega_s$). For $T \sim \omega_s$ the expression (4.10) for P_1 is applicable provided $\ln(\omega_c/\tilde{\omega}) \gg 1$.

Only the virtual transitions where low-frequency ($\tilde{\omega} \leqslant \omega_{k\alpha} \leqslant \omega_c$) WS phonons are created or annihilated contribute to P_1. Those transitions where high-frequency phonons ($\omega_{k\alpha} \gg \omega_c$) participate are inessential (their contribution is of order $(\omega_c^2/\omega_p^2)P \ll P$), since the corresponding terms in Re $\Pi_1^{(1)}(\omega_c + i0)$ and Re $\Pi_1^{(1)}(+i0)$ compensate one another. A similar compensation occurs for multiphonon transitions. This provides rapid convergence of the expansion of P in a number of phonons participating in a virtual transition, because the density of states of the low-frequency WS phonons contributing to P is small.

It is obvious from equations (4.7) and (4.10) that the CR peak shift P exceeds substantially the halfwidth Γ for $\omega_p \gg \omega_c$.

4.4. The corrections to the Born approximation

The WS phonons contributing to the damping and frequency shift of the phonon $\{k1\}$ ($k \to 0$) are themselves renormalised due to the coupling to a medium. If this renormalisation is strong, it should be taken into account when Γ and P are calculated.

The self-energy $\Pi_{k\alpha}(\omega)$ of an arbitrary WS phonon $\{k\alpha\}$ may be calculated in a way similar to that used for the phonon $\{k1\}$ with $k \to 0$ (cf Dykman (1982) where the case $H_\perp = 0$ was considered). In the Born approximation $\Pi_{k\alpha}(\omega)$ is the sum of two addends analogous to $\Pi^{(1)}(\omega)$, $P^{(2)}$. The main difference lies in the coefficient l^2/NS that for $\Pi_{k\alpha}(\omega)$ should be replaced by $|A_{k\alpha}|^2$. Since $|A_{k,-1}|^2$ increases strongly with decreasing k (cf equations (4.3) and (2.2)) the renormalisation of low-lying WS phonons appears to be large. At $\omega_p \gg \omega \gg \omega_s$, when the 'single-site approximation' (2.18) is valid, the main term in $\Pi_{k\alpha}(\omega)$ is analogous to P_0 in equation (4.10):

$$\Pi_{k\alpha}(\omega) \simeq P_{k\alpha} \qquad \omega_p \gg \omega \gg \omega_s$$

$$P_{k\alpha} = NS|A_{k\alpha}|^2 \sum_q q^2 |V_q|^2 \omega_q^{-1} \exp(-\eta q^2). \tag{4.11}$$

It is convenient to single out the frequency Ω_0 of those phonons whose 'bare' frequency $\omega_{k\alpha}$ equals the shift $P_{k\alpha}$:

$$\Omega_0 \equiv \omega_{k_0,-1} \qquad \omega_{k_0,-1} = P_{k_0,-1}. \tag{4.12}$$

According to equations (2.2), (4.3) and (4.11) $\Omega_0 \simeq \frac{1}{2}(P\omega_c)^{1/2}$ at $P \gg \omega_c^3/\omega_p^2$, and $\Omega_0 \sim (P^3\omega_p^2/\omega_c)^{1/4}$ at $P \ll \omega_c^3/\omega_p^2$. Obviously the renormalisation of the phonons with frequencies $\omega_{k\alpha} \lesssim \Omega_0$ is strong. However this renormalisation does not influence the applicability of the decoupling resulting in equations (2.10) and (2.11), since the contribution of the low-frequency ($\omega_{k\alpha} \lesssim \Omega_0 \ll \omega_c$) phonons to $\Pi^{(1)}(\omega)$ is small ($\sim \Omega_0/\omega_p$) for $\omega \sim \omega_c \gg \Omega_0$ (the decoupling concerned $\Pi^{(1)}(\omega)$ only). If $\Omega_0 \gtrsim \tilde{\omega}$ at the same time the explicit form of $P^{(2)}$ alters.

To obtain $P^{(2)}$ one should use the exact expression (2.14). The result, however, may be understood from qualitative considerations. Indeed, it is obvious from equation (4.11) that the coupling to MV causes the dynamical 'hardening' of the low-lying WS phonons. Therefore the low-frequency cut-off parameter $\tilde{\omega}$ for the phonons contributing to $P^{(2)}$ should be redefined:

$$\tilde{\omega} = \max(\omega_s, T, \Omega_0). \tag{4.13}$$

The rigorous calculation (similar to that carried out at $H_\perp = 0$ (Dykman 1982)) shows that the expression (4.10) for P with $\tilde{\omega}$ given by (4.13) is valid for $\ln(\omega_c/\tilde{\omega}) \gg 1$.

Although the term P_1 in P renormalised at $\Omega_0 \gg \omega_s$, T is small compared with P_0, the renormalisation of P_1 is essential, since this term exceeds the halfwidth Γ of the CR peak.

5. CR peak halfwidth and shift for a simple coupling model

The expressions for Γ and P obtained above simplify if

$$|V_q|^2 \omega_q^{-1} = S^{-1}\mathcal{D}q^{2n}. \tag{5.1}$$

This model is actual: for example, the deformation potential approximation corresponds to $n = 0$; for electrons above the liquid helium $n = -1$ at large pressing fields (see below).

According to equations (3.4), (4.7) and (5.1) at low densities

$$\Gamma = \frac{\mathcal{D}T\tau_e}{2^{3/2}\pi}(\tfrac{1}{2}l^2)^{-(n+1)}(n+\tfrac{3}{2})! \qquad T \gg \tau_e^{-1} \gg \omega_s \qquad \left(x! \equiv \int_0^\infty dy\, y^x e^{-y}\right) \tag{5.2}$$

while at high densities and relatively high temperatures

$$\Gamma \simeq \Gamma_1 = \frac{\mathcal{D}l^2 T}{32\pi m c_t^2 N}\eta^{-(n+3)}(n+2)! \qquad \omega_p \gg \omega_c > T \gg \omega_s. \tag{5.3}$$

If follows from equations (3.3), (4.4), (5.2) and (5.3) that $\Gamma \propto \mathcal{D}T^{1/2}N^{-3/4}H_\perp^{n+3/2}$ at low densities and $\Gamma \propto \mathcal{D}TN^{3(n+1)/4}H_\perp^{-1}$ at high densities. These expressions depend on N in qualitatively different ways (the derivatives are opposite in sign) for $n > -1$; this is also the case for their dependence on $H_\perp$.

The shift P of the CR peak at low densities according to equations (3.5) and (5.1) is given by

$$P \simeq P_0 + P_T \qquad T \gg \tau_e^{-1} \gg \omega_s$$

$$P_0 = -\frac{\mathcal{D}}{4\pi}(\tfrac{1}{2}l^2)^{-(n+1)}(n+1)(n+1)!$$

$$P_T = \frac{\mathcal{D}T}{2\pi\omega_c}(\tfrac{1}{2}l^2)^{-(n+1)}\left(\tfrac{3}{2}(n+2)! - 2\sum_{m=1}^{\infty}\frac{(n+m+2)!}{m(m+1)(m+2)!}\right) \tag{5.4}$$

(the terms of order $\omega_s^2\tau_e^2$ are neglected here). We suppose that $n < 1$ in equation (5.1) (otherwise the interaction (5.1) should be cut off at large q in order to become compatible with the assumption of weak CR spectrum renormalisation). Then the series (5.4) converges.

At high densities and $T \gg \omega_s$ equations (4.10) and (5.1) yield

$$P \simeq P_0 + P_1 \qquad \omega_p \gg \omega_c \gg \tilde{\omega}$$

$$P_0 = \frac{1}{4\pi}\mathcal{D}l^2\eta^{-(n-2)}(n+1)!$$

$$P_1 = \frac{\mathcal{D}Tl^2}{16\pi^2 m c_t^2 N}\ln(\omega_c/\tilde{\omega})\eta^{-(n-3)}(n+2)! \qquad T \gg \omega_s. \tag{5.5}$$

The addend P_0 in equations (5.4) and (5.5) is independent of T. At low densities $P_0 \propto \mathcal{D}H_\perp^{n+1}$, $P_T \propto \mathcal{D}TH_\perp^n$ and P is independent of N. At high densities $P_0 \propto \mathcal{D}N^{3(n-2)/4}H_\perp^{-1}$ and $P_1 \propto \mathcal{D}TN^{3(n-1)/4}H_\perp^{-1}\ln(\omega_c/\tilde{\omega})$.

It is obvious that the dependences of Γ and P on N and $H_\perp$ are rather specific and sensitive to the exponent n in equation (5.1). Therefore CR may be used to investigate both the many-electron effects and the type of coupling to MV. Numerical examples of the application of the results presented above to WS on the helium surface are given in detail in § 6.

5.1. Generalisation on the case of scattering by impurities

For a typical value of the electron density $N \approx 3 \times 10^{11}$ cm^{-2} compatible with Wigner crystallisation in the semiconductor surface inversion layer and $H_\perp \approx 50$ kOe we obtain for the system with Si parameters $\omega_p/\omega_c \geqslant 2$ and

$$\frac{\omega_s}{\omega_c} \sim \left(\frac{mc_s^2}{\hbar\omega_c}\right)^{1/2}\left(1 + \frac{\omega_p^2}{\omega_c^2}\right)^{1/4} \leqslant 0.1$$

(c_s is the semiconductor sound velocity). Then equation (5.3) should be used to calculate the CR peak halfwidth. It yields $\Gamma \sim 10^{11}\ s^{-1}$ for $T = 10\,K$ and an effective (cf Dykman 1978) deformation potential constant 7 eV. The value of Γ increases rapidly with softening of surface phonons or increasing coupling.

The Wigner crystal on the semiconductor surface is coupled not only to MV but also to impurities. The random field of impurities (in contrast to that of MV (Dykman 1981)) results in pinning of WS. Pinning in a weak field is connected with the transverse deformation of the WS and takes place at arbitrary field intensity. For a weak random potential with a short ($\lesssim \eta^{1/2}$) correlation range the correlation length of the pinned WS r_{corr} may be expressed in terms of the single-electron momentum relaxation time $\tau(\tilde{\pi})$ calculated for $H_\perp = 0$ ($\tilde{\boldsymbol{\pi}}$ is the electron momentum): $r_{corr} \sim N^{-1/2}(mc_t^2\tau(\eta^{-1/2}))^{1/2}$ for $T \ll \omega_p, \omega_p^2/\omega_c$ (cf Dykman 1981 where the case $H_\perp = 0$ was consisered.

Pinning causes an uncertainty of order r_{corr}^{-1} in the WS phonon momentum and thus results in an inhomogeneous broadening and shift of the CR peak of order $\omega_p^2(\omega_c r_{corr} N^{1/2})^{-1}$ (cf equation (2.2)), which as a rule exceeds the homogeneous (decay) CR broadening caused by impurities. The latter is given by equations (3.4) and (4.7) with $2|V_q|^2 T/\omega_q$ and $|V_q|^2(2\bar{n}_q + 1)$ replaced by the Fourier component of the random field correlator.

6. CR of electrons above the liquid helium surface

The relaxation of electrons above the helium surface for $T < 1\,K$ is caused by the coupling to ripplons (quantised waves on the helium surface). The capillary surface waves are slow: for electron densities $N \lesssim 2 \times 10^9\ cm^{-2}$ (at higher N the charged helium surface becomes unstable) and for reasonable $H_\perp$ ($H_\perp \lesssim 20$ kOe) the energy $\hbar\omega_s$ (in temperature units) does not exceed $5 \times 10^{-3}\,K$. Therefore the scattering by ripplons is usually quasi-elastic and we shall suppose $T \gg \hbar\omega_s$. Then the coupling parameters V_q and ripplon eigenfrequencies ω_q enter the expressions for Γ and P only in the combination $|V_q|^2/\hbar\omega_q$. Using the theory of Shikin and Monarkha (1974) one obtains for $q \gg 10^2\ cm^{-1}$

$$\frac{|V_q|^2}{\hbar\omega_q} = S^{-1}\frac{e^2}{2\alpha_s q^2}[E_\perp + \tfrac{1}{2}E_H l^2 q^2(1 + \ln(q/4\gamma))]^2 \qquad q \ll 4\gamma$$

$$E_H = \gamma a_B e/l^2 \qquad a_B = \hbar^2/me^2. \tag{6.1}$$

Here α_s is the surface tension, $\alpha_s = 0.35\ dyn\,cm^{-1}$, γ^{-1} is the length of electron localisation in the direction normal to the surface and $E_\perp$ is the electric field 'pressing' electrons to the surface. The latter determines the limiting electron density:

$$N_0(E_\perp) = E_\perp/2\pi e. \tag{6.2}$$

The characteristic field E_H in (6.1) is small ($E_H \lesssim 300\ V\,cm^{-1}$ for $H_\perp \lesssim 20$ kOe), so the term proportional to E_H is essential only at low $E_\perp$.

6.1. CR spectrum in the low-density range

For $\omega_c > T/\hbar \gg \omega_p^2/\omega_c$ equations (3.4), (3.5) and (6.1) yield the following expressions for

the halfwidth and temperature-independent part of the shift of the CR peak:

$$\Gamma = \frac{T\tau_e e^2}{8(2\pi)^{1/2}\hbar^2\alpha_s}[E_\perp^2 + 3E_\perp E_H(\xi - 1.7) + \tfrac{15}{4}E_H^2(\xi^2 - 3.8\xi + 3.7)]$$

$$P_0 = -\frac{e^2 E_H}{4\pi\hbar\alpha_s}[E_\perp(\xi - 2.1) + 2E_H(\xi^2 - 4.1\xi + 4.3)] \qquad T \gg \hbar/\tau_e \qquad \xi = \ln(4\gamma l) \tag{6.3}$$

(here the numerical coefficients are given to an accuracy of two decimal places; their explicit form is tedious).

The expression for P_T is also quadratic in $E_\perp$, the term proportional to $E_\perp^2$ being $e^2E_\perp^2 T/4\pi\hbar^2\alpha_s\omega_c$ (cf Dykman and Khazan 1979). To calculate the term proportional to E_H^2 in P_T one should take into account that the expresssion (6.1) for $|V_q|^2/\hbar\omega_q$ is cut off at $q \sim 4\gamma$. If $\xi \gg 1$ (the electrons may be assumed to be two dimensional only for $4\gamma l \gg 1$) the term proportional to E_H^2 is negative: it is proportional to $-(Te^2E_H^2/\hbar^2\alpha_s\omega_c)\xi^3$. Therefore for large ξ the CR peak shift $P \approx P_0 + P_T$ changes sign depending on $E_\perp$: $P < 0$ for $E_\perp \ll E_H\xi\hbar\omega_c/T$ or $E_\perp \ll E_H\xi^{3/2}$, and $P > 0$ at higher $E_\perp$. For any $E_\perp/E_H$ in the low-density range $P \ll \Gamma$.

The parameter Γ at constant N increases monotonously with $E_\perp$. If, however, the helium surface is charged up to saturation, $N = N_0(E_\perp)$, the dependence of $\Gamma = \Gamma_{sat}$ on $E_\perp$ has a minimum. This is obvious if we rewrite the expression (6.3) for Γ_{sat} taking equations (3.3) and (6.2) into account:

$$\Gamma_{sat} = \psi'(T, \omega_c)\Gamma'(E_\perp/E_H) \qquad T \gg \hbar/\tau_e \qquad \psi'(T, \omega_c) \simeq 0.12(\gamma^{(0)}a_B)^{5/4}\frac{T^{1/2}(e^2/l)^{3/2}}{\hbar\alpha_s l^2}$$

$$\Gamma'(x) = (\gamma/\gamma^{(0)})^{5/4}x^{-3/4}[x^2 + 3x(\xi - 1.7) + \tfrac{15}{4}(\xi^2 - 3.8\xi + 3.7)] \tag{6.4a}$$

$$(x \equiv E_\perp/E_H).$$

Here $\gamma^{(0)}$ is the value of γ at $E_\perp = 0$, $\gamma^{(0)} = 1.3 \times 10^6\,\mathrm{cm}^{-1}$. To first order in $E_\perp$

$$\gamma \equiv \gamma(E_\perp/E_H) \simeq (1 + z_\perp)\gamma^{(0)} \qquad z_\perp = \tfrac{3}{2}(\gamma^{(0)}l)^{-2}E_\perp/E_H^{(0)} \qquad E_H^{(0)} = \gamma^{(0)}a_B e/l^2 \tag{6.4b}$$

($z_\perp$ is small for $E_\perp \ll E_H^{(0)}$, since $(\gamma^{(0)}l)^2 \gg 1$).

The dependence of Γ_{sat} on $E_\perp$ is described by $\Gamma'(E_\perp/E_H)$. It is obviously non-monotonic with a minimum at $E_\perp/E_H \simeq 1.5\xi - 2.9$ (for $\xi \gg 1$). Such a dependence is specific for the many-electron system under consideration. It was found experimentally by Edel'man (1979). The data were obtained at such magnetic fields and temperatures that $T/\hbar\omega_c = 0.45$–0.49, $\gamma^{(0)}l \simeq 4.1$, $E_H^{(0)} \simeq 100\,\mathrm{V\,cm}^{-1}$ and $\psi'(T, \omega_c)/\omega_c \simeq (8.1$–$8.4) \times 10^{-5}$ ($\psi'(T, \omega_c) \sim 10^7\,\mathrm{s}^{-1}$). The lowest value of $E_\perp$ corresponded to $E_\perp/E_H^{(0)} \sim 0.1$.

Equations (6.3), (6.4*a*) and (6.4*b*) describe the experimental data both qualitatively and quantitatively without any adjustable parameters. This is obvious from figure 1. The experimental error in the range $0.1 \leqslant E_\perp/E_H^{(0)} \leqslant 0.4$ is about equal to the discrepancy between theory and experiment. At higher $E_\perp$ the discrepancy grows, but up to $E_\perp/E_H^{(0)} \sim 1$ (in particular, near the minimum of $\Gamma'(E_\perp/E_H)$) remains smaller than 40% (the spread in experimental data also increases in this range of $E_\perp$). For $E_\perp/E_H^{(0)} > 1.2$ the disagreement with experiment becomes appreciable; the parameter $T\tau_e/\hbar$ is too small here, $T\tau_e/\hbar < 2.3$, and equation (6.3) is inapplicable. The shift of the CR peak in the range $E_\perp/E_H^{(0)} < 1.2$ was small in Edel'man's experiment, in agreement with the predictions of the present theory.

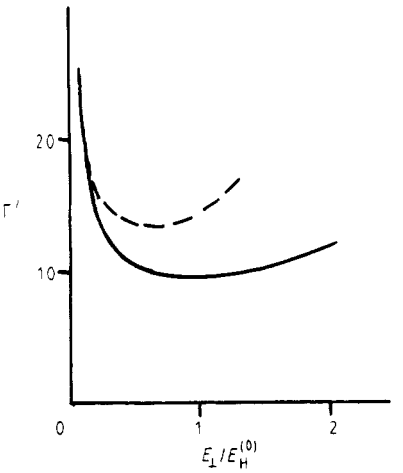

Figure 1. The dependence of the reduced CR peak halfwidth Γ' on $E_\perp$ in the low-density range for $\gamma^{(0)}l = 4.1$. The broken curve shows Edel'man's (1979) experimental results for the range $E_\perp/E_H^{(0)} \leqslant 1.2$.

6.2. Halfwidth and shift of the CR peak in the high-density range

In the range $\omega_p \gg \omega_c$ the term proportional to E_H in equation (6.1) may be neglected (the inequality $E_\perp \gg E_H$ follows from $\omega_p \gg \omega_c$ for $H_\perp > 100$ Oe). Then Γ and P are given by equations (5.3) and (5.5) with $n = -1$, $\mathcal{D} = e^2 E_\perp^2/2\alpha_s$:

$$\Gamma \approx \Gamma_1 = \frac{e^2E_\perp^2 T}{16\pi\hbar^2\alpha_s\omega_c\mu^2}\frac{\omega_p^2}{c_t^2N}$$

$$P = P_0 + P_1$$

$$P_0 = \frac{e^2E_\perp^2\omega_p}{4\pi\hbar\alpha_s\omega_c\mu}$$

$$P_1 = \frac{e^2E_\perp^2 T\ln(\omega_c/\tilde{\omega})}{8\pi^2\hbar^2\alpha_s\omega_c\mu^2}\frac{\omega_p^2}{c_t^2N} \qquad \mu = 2\eta m\omega_p/\hbar,\ \omega_p \gg \omega_c > T \gg \omega_s \qquad (6.5)$$

(for the hexagonal WS parameter ω_p^2/c_t^2N equals about 25.6 (Bonsall and Maradudin 1977)). Since μ is constant for $\omega_p \gg \omega_c$, the halfwidth Γ_1 does not depend on the density N (as for an ideal electron gas). The term P_0, dominating in the expression for the shift P for $\omega_p \gg \omega_c \gg T$, is proportional to $N^{3/4}$ (Shikin (1979) approximated the CR peak shift empirically at arbitrary ω_p/ω_c by the expression (6.5) for P_0 divided by two assuming μ to be dependent on the magnetic field). With an accuracy up to a logarithmic factor both Γ and P are proportional to $H_\perp^{-1}$.

The expressions (6.5) for Γ and P agree qualitatively with Edel'man's experimental data for the range $\omega_p \gtrsim 1.5\,\omega_c$. The latter are obtained only for the saturation regime $N = N_0(E_\perp)$. The quantitative agreement with experiment is achieved for Γ_{sat} at $\mu = 1$. However, such a value of μ is too small: in the Debye model $\mu \approx 1.8$. But the latter value is too large, since it ignores the essential hardening of transverse WS phonons in the short-wavelength range. Substituting the transverse sound velocity obtained by averaging the numerical dispersion law (given by Bonsall and Maradudin) over the Brillouin

zone into the expression for μ appropriate for the Debye model one finds $\mu \simeq 1.4$ (this estimate is certainly crude).

In the experimental conditions $\omega_c/\omega_p \geqslant 0.4$. Therefore the corrections to Γ_{sat} due to decays into several WS phonons may be essential. In particular, the two-phonon decays according to (4.8) and (6.1) yield

$$\Gamma_2 = \Gamma_1 \frac{\omega_c}{\omega_p} \frac{\omega_p^2}{4\pi\mu c_t^2 N}. \tag{6.6}$$

To compare $\Gamma_{sat} = \Gamma_1 + \Gamma_2$ and P_{sat} with experiment it is convenient to write them in the following form:

$$\Gamma_{sat} = \psi''(T, \omega_c)\Gamma''(E_\perp/E_c)$$

$$\psi''(T, \omega_c) \simeq 0.26e^2E_c^2T(\hbar^2\alpha_s\omega_c)^{-1} \qquad \Gamma''(x) = x^2(1 + 1.5x^{-3/4})$$

$$P_{sat} = \chi(\omega_c)P''(E_\perp/E_c, T/\hbar\omega_c)$$

$$\chi(\omega_c) \simeq 0.057e^2E_c^2(\hbar\alpha_s)^{-1} \qquad P''(x, T/\hbar\omega_c) = x^{11/4} + 2.9(T/\hbar\omega_c)x^2\ln(\omega_c/\tilde{\omega})$$

$$E_c = (2\pi\hbar\omega_c a_B e^{-2})^{1/3}e/l^2 \qquad x \equiv E_\perp/E_c,\ \omega_p \gg \omega_c \tag{6.7}$$

(the numerical coefficients are evaluated here for $\mu = 1.4$, $\omega_p^2/c_t^2N = 25.6$; E_c denotes the value of $E_\perp$ corresponding to a value of $N_0(E_\perp)$ such that $\omega_p = \omega_c$).

The 'high-density' experimental data were obtained by Edel'man for $E_c \simeq 3.7 \times 10^2$ V cm^{-1}, $0.85 \geqslant T/\hbar\omega_c \geqslant 0.45$, $\psi''(T, \omega_c)/\omega_c \simeq (2.2 \times 10^{-3})T/\hbar\omega_c$ ($\psi'' \sim 10^8$ s^{-1}) and $\chi(\omega_c)/\omega_c \approx 4.8 \times 10^{-4}$($\chi(\omega_c) \simeq 5.6 \times 10^7$ s^{-1}). The parameter P_{sat}/ω_c is extremely small for actual $E_\perp/E_c \leqslant 5$; therefore $T \gg \hbar(P_{sat}^3\omega_p^2/\omega_c)^{1/4}$ and according to (4.13) $\tilde{\omega} = T/\hbar$ in equation (6.7).

It is obvious from figure 2 that equation (6.7) describes well the experimental dependence of both Γ_{sat} and P_{sat} on $E_\perp$ (the discrepancy is less than 20%). The qualitative consequences of (6.7), the proportionality of Γ_{sat} to T and the negativeness of

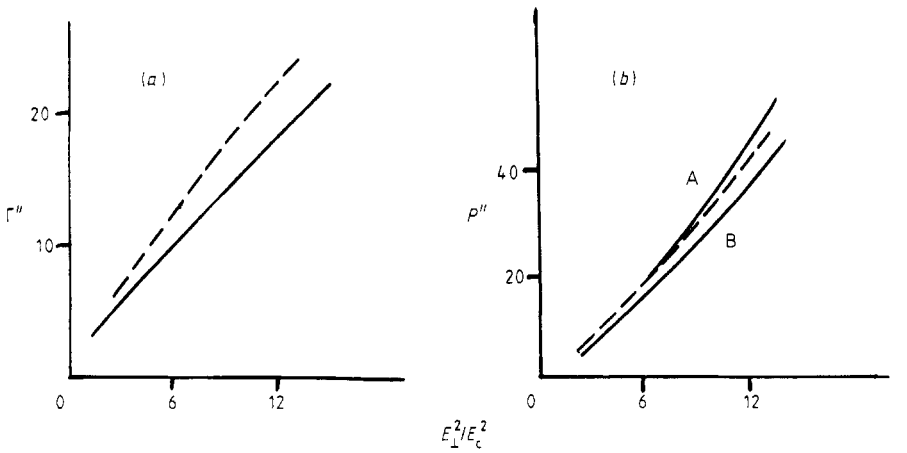

Figure 2. The dependence of the reduced CR peak halfwidth Γ'' (*a*) and shift P'' (*b*) on $E_\perp$ in the high-density range ($E_\perp/E_c \geqslant 1.7$). The broken curves show the experimental results (Edel'man 1979). The curves A and B in (*b*) correspond to $T/\hbar\omega_c = 0.45$ and 0.84 (the limiting values of $T/\hbar\omega_c$ investigated experimentally). For $(E_\perp/E_c)^2 < 6$ curve A practically coincides with the broken curve and is therefore not shown.

$\partial^2\Gamma_{sat}/\partial(E_\perp^2)^2$, are confirmed by experiment. The experimental dependence of P_{sat} on T was not observed. Although T enters the expression (6.7) for P_{sat} explicitly, the dependence of P_{sat} on T appears to be weak numerically, and P_{sat} is approximately proportional to $E_\perp^2$ under the experimental conditions (at the same time the temperature-dependent part of P_{sat} is not small; its contribution is about 25%).

It should be noted that for the whole range of $E_\perp$, T investigated by Edel'man the CR broadening Γ exceeds ω_s and thus the single-site approximation (2.17) and (2.18) is valid with a high accuracy.

7. Conclusions

It follows from the results of the present paper that the CR peak shape for a Wigner solid coupled to a medium is Lorentzian. In the actual case of quasi-elastic scattering, $\omega_c \gg \omega_s$, the broadening and shift of the peak may be calculated within the single-site approximation (2.18), i.e. neglecting the interference of short-wavelength MV scattered by different electrons. Such an approach differs from that used by Eguiluz *et al* (1981) to investigate the structure of the conductivity $\sigma(\boldsymbol{k}, \omega)$ at $\omega \simeq \omega_q$ with q close to the WS reciprocal lattice vectors; in the present paper we have investigated $\sigma(\boldsymbol{k}, \omega)$ at $\omega \sim \omega_c \gg \omega_s$, where this structure is smeared.

At low densities or high magnetic fields where $\omega_p \ll \omega_c$ and $T \gg \hbar\omega_p^2/\omega_c$ the shift P and halfwidth Γ of the CR peak are determined by the short-range order in the electron system. The CR broadening is due to the decays of the phonon $\{\boldsymbol{k}1\}, k \to 0$ (it is responsible for resonant light absorption) where an MV quantum and many short-wavelength WS phonons participate; $\Gamma \gg P$ here. The results obtained in this range are valid not only for a WS but also for a strongly correlated electron liquid.

At high densities, $\omega_p \gg \omega_c$, the transverse WS phonons with relatively long wavelength contribute to Γ and P. In particular the CR broadening is due to decays of the phonon $\{\boldsymbol{k}1\}$ $(k \to 0)$ into the phonon $\{\boldsymbol{k}', -1\}$ $(k' \simeq \omega_c/c_t)$ and an MV quantum. Thus the long-range ordering is apparently manifest. At high densities $P \gg \Gamma$, and the corrections to the Born approximation for P may exceed Γ.

The good qualitative and quantitative agreement of the present theory, which in practice does not contain adjustable parameters, with the experiment confirms the essential role of many-electron effects in scattering by MV. At the same time it becomes clear that in quantising magnetic fields rigid short-range order exists up to low densities: $e^2(\pi N)^{1/2}/T \sim 30$. The value of the transverse sound velocity $c_t \simeq 0.2\omega_p N^{-1/2}$ obtained for a WS numerically by Bonsall and Maradudin (1977) appears to be compatible with experiment. Thus the specific dependences of the CR peak halfwidth and shift on the electron density, temperature and so on (parallel with traditional applications) permit us to use CR to detect both the short-range order in a system of 2D electrons (applying relatively strong magnetic fields) and the long-range one (applying relatively weak magnetic fields).

Appendix

The polarisation operator $\Pi^{(1)}(\omega)$ (2.11) may be written as the sum over lattice sites:

$$\Pi^{(1)}(\omega) = \sum_n \Pi^{(1)}(\omega, n) \qquad \Pi^{(1)}(\omega, n) = -\tfrac{1}{2}\mathrm{i}\sum_q l^2q^2|V_q|^2 \int_0^\infty \mathrm{d}t\, \exp(\mathrm{i}\omega t)\Pi_{qn}(t)$$

$$\Pi_{qn}(t) = \zeta_{qn}(t)\varphi(\omega_q, t) - \mathrm{cc} \qquad \zeta_{qn}(\mathrm{t}) = \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{R}_n + W_q(\boldsymbol{R}_n, t)). \tag{A.1}$$

To estimate $\Pi^{(1)}(\omega, n)$ for $R_n \neq 0$ it is convenient to single out in $\zeta_{qn}(t)$ the factor describing its oscillation and damping with q:

$$\zeta_{qn}(t) = \mathfrak{D}_n(\boldsymbol{q}, t) \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{R}_n - q^2 q_{sn}^{-2}) \qquad R_n \neq 0. \tag{A.2}$$

According to equation (2.16) $\mathfrak{D}_n(\boldsymbol{q}, t)$ is a smooth function of q and $q_{sn}^2 \leqslant (\Sigma_{k\alpha} |A_{k\alpha}|^2)^{-1}$. If the characteristic range t_{sn} of integration over t in the expression for $\Pi^{(1)}(\omega, n)$ is not too large, so that

$$R_n^{-1} |\partial\chi(q, t)/\partial q| \ll |\chi(q, t)|$$

for $t \leqslant t_{sn}$, $q \leqslant q_{sn}$ $(\chi(q, t) = q^2 |V_q|^2 \varphi(\omega_q, t))$, (A.3)

then the coefficient at $\zeta_{qn}(t)$ in equation (A.1) is a smooth function of q compared wtih $\exp(\mathrm{i}\boldsymbol{q}\boldsymbol{R}_n)$, and the integration over $\boldsymbol{q}$ in (A.1) gives a small value for $\Pi^{(1)}(\omega, n)$. In particular, in the actual case when $\chi(q, t)$ is not only smooth in the scale R_n^{-1} but varies slowly in the whole range $q \leqslant q_{sn}$,

$$q |\partial\chi(q, t)/\partial q| \leqslant |\chi(q, t)| \qquad q \leqslant q_{sn}, t \leqslant t_{sn}, \tag{A.4}$$

the term $\Pi^{(1)}(\omega, n)$ is exponentially small:

$$\Pi^{(1)}(\omega, n) \sim \Pi^{(1)}(\omega, 0) \exp(-\tfrac{1}{4} q_{sn}^2 R_n^2) \qquad (q_{sn} R_n \gg 1). \tag{A.5}$$

Since q_{sn}^{-1} substantially exceeds the atomic spacing in the medium, the functions $q^2 |V_q|^2$ and $q^2 |V_q|^2/\omega_q$ are smooth in the range $q \leqslant q_{sn}$. Consequently the condition (A.4) will be fulfilled surely if

$$\omega_q t_{sn} \ll 1 \qquad \text{for } q \leqslant q_{sn}. \tag{A.6}$$

For the few lowest R_n the integration over t in $\Pi^{(1)}(\omega, n)$ is similar to that carried out in §§3 and 4 for $\Pi^{(1)}(\omega, 0)$, so that $t_{sn} \sim t_s \equiv t_{s0}$, $q_{sn} \sim q_s \equiv q_{s0}$ and (A.6) reduces to the inequality $\omega_s t_s \ll 1$ used in §§ 3 and 4.

For large R_n the summation over $\boldsymbol{k}$ in the time-dependent terms in $W_q(\boldsymbol{R}_n, t)$ (see equation (2.16)) is limited practically to small $k \leqslant k_n = R_n^{-1}$, and thus effective 'bands' for the phonons contributing to these terms are narrow: $\omega_{k\alpha} \leqslant \omega_{k_n\alpha}$. The main contribution to $\Pi^{(1)}(\omega, n)$ for $\omega \sim \omega_c \gg \omega_{q_{sn}}$ then results from the resonant term in the expansion of $\zeta_{qn}(t)$,

$$\zeta_{qn}^{(r)}(t) = \sum_{k} |\boldsymbol{q}\boldsymbol{A}_{k1}|^2 \exp(-\mathrm{i}\omega_{k1} t - \mathrm{i}\boldsymbol{k}\boldsymbol{R}_n) \exp(-q^2 q_{sn}^{-2} + \mathrm{i}\boldsymbol{q}\boldsymbol{R}_n). \tag{A.7}$$

This term describes the decays of the phonon under consideration (i.e. the phonon $\{\boldsymbol{k}1\}$, $k \to 0$) into another long-wavelength ws phonon of the branch $\alpha = 1$. It follows from equations (A.1) and (2.2) that t_{sn} is limited here to $|\omega_{k_n 1} - \omega_c|^{-1} \sim \omega_c R_n N^{1/2}/\omega_p^2$ (this allows us to neglect the time dependence of the contribution to $\zeta_{qn}^{(r)}(t)$ of the phonons with $\alpha = -1$, since $t_{sn}\omega_{k_n,-1} \sim (R_n N^{1/2})^{-1/2} \ll 1$). There is, however, another limitation on t_{sn} for large $R_n N^{1/2}$. Indeed, to estimate $\Pi^{(1)}(\omega, n)$ for $R_n \neq 0$ by perturbation theory we should replace the 'bare' frequencies ω_{k1} at small k in equation (A.7) by $\omega_{k1} + \Pi^{(1)}(\omega_{k1} + \mathrm{i}0, 0) \equiv P^{(2)} \simeq \omega_{k1} + P - \mathrm{i}\Gamma$. Then $t_{sn} \leqslant \Gamma^{-1}$, $\omega_c R_n N^{1/2}/\omega_p^2$ and (A.3) is fulfilled for smooth $q^2 |V_q|^2/\omega_q$ and large R_n provided

$$|\omega_{q + R_n^{-1}} - \omega_q| \leqslant \max(\Gamma, \omega_p^2(\omega_c R_n N^{1/2})^{-1}) \qquad q \leqslant q_{sn}. \tag{A.8}$$

It follows from the results obtained in § 6 that for 2D electrons above the liquid helium surface in the actual range of temperatures, densities and quantising magnetic

field strengths $\omega_q \ll \Gamma$ for $q \leqslant q_s$. Then inequalities (A.8) and (A.6) are obviously fulfilled and the exponential estimate (A.5) is valid for $\Pi^{(1)}(\omega, n)$ at arbitrary R_n. This estimate also holds for typical semiconductor devices (cf § 5).

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