Noise-enhanced optical heterodyning in an all-optical bistable system

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A new form of frequency-selective all-optical heterodyning at the difference frequency of two modulated laser beams, related to stochastic resonance, is reported. A noise-induced enhancement of the heterodyne signal and of the signal-to-noise ratio has been predicted and observed in an all-optical bistable system for the beams at different wavelengths. © *1995 American Institute of Physics.*

Recent progress in optical signal processing and optical communication[1,2] has highlighted the problem of controlling the signal and the signal-to-noise ratio (SNR) in optical systems. An interesting effect that may provide a new and effective tool for such control is stochastic resonance (SR) in which both the signal and the SNR at the output of a nonlinear system can be *increased* by adding noise.[3] Most investigations of SR to date[4] have related to low-frequency signals driving bistable systems, including active[5] and passive[6,7] optically bistable (OB) systems. In such cases, the SR mechanism can operate effectively provided (i) the stationary populations of the states are nearly equal to each other,[8] and (ii) the frequency of the force is much smaller than the reciprocal relaxation time τ_r^{-1} of the system. It was recently suggested,[9] however, and demonstrated in analogue simulations, that a related phenomenon can occur when a nonlinear system is driven by two *high frequency* signals: if the resultant heterodyne signal is of sufficiently low frequency, both it and its SNR can be enhanced by the addition of noise.

In this letter we report observation and outline a theory of a new form of all-optical heterodyning, *noise-enhanced optical heterodyning* (NEOH). The system studied is a bistable interferometer with a dispersive mechanism of nonlinearity. The intensity of the light at the output of the interferometer I_T is related to that of the incident light I_{in} via a transmission coefficient $N(\phi)$ that depends periodically on the phase gain ϕ of the light inside the interferometer,

$$I_T=I_{\text{in}}\,N(\phi),\quad N(\phi+2\pi)=N(\phi). \tag{1}$$

The phase gain itself depends nonlinearly on I_{in}. A practically important example of a two-dimensional array of bistable microresonators, of 4 μm diameter and threshold power below 100 μW, has been demonstrated recently.[10]

There are several possible schemes for noise-enhanced optical heterodyning. We consider one here for which, in addition to the resonant signal beam, the cavity is driven by a nonresonant reference beam of intensity $I_{\text{ref}}(t)$, which affects the phase gain ϕ for the resonant beam. The dynamics of the phase gain in such a system can often[11] be described, to a reasonable approximation, by a Debye relaxation equation

$$\dot\phi+\frac{1}{\tau_r}(\phi-\phi_0)=I_{\text{in}}(t)\;M(\phi)+I_{\text{ref}}(t),$$
$$M(\phi+2\pi)=M(\phi), \tag{2}$$

where ϕ_0 is the phase of the interferometer in the dark (I_{in}, I_{ref} are assumed to be properly scaled). The explicit forms of the periodic absorption coefficient $M(\phi)$, and of $N(\phi)$, depend on the construction of the interferometer and are well-known for simple models, such as a Fabry-Perot cavity filled with a medium with cubic nonlinearity (cf. Ref. 11).

We assume that the intensity of the resonant radiation has a high-frequency component (of frequency $\omega_0\gg\tau_r^{-1}$) with amplitude $A_{\text{in}}(t)$ and/or phase $\psi(t)$ modulated by a low-frequency signal, whereas the intensity of the reference beam includes the high-frequency component and the noise $\Delta I(t)$,

$$I_{\text{in}}(t)=\overline{I_{\text{in}}}+A_{\text{in}}(t)\cos[\omega_0 t+\psi(t)],$$
$$I_{\text{ref}}(t)=\overline{I_{\text{ref}}}+A_{\text{ref}}\cos\omega_0 t+\Delta I(t), \tag{3}$$
$$\langle\Delta I(t)\Delta I(t')\rangle\approx 2D\,\delta(t-t').$$

Here, $\overline{I_{\text{ref}}}$ and $\overline{I_{\text{in}}}$ are constant components in the intensities of the two laser beams.

Equations (2) and (3) describe Brownian motion of the phase gain. In the case where the characteristic modulation frequency is small, $|\dot\psi|,|\dot A_{\text{in}}|/A_{\text{in}}\ll\tau_r^{-1}\ll\omega_0$, this consists of a comparatively slow motion, $\phi^{(\text{sl})}(t)$, with superimposed fast oscillations at frequency ω_0. Because of the nonlinearity

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of $M(\phi)$ the oscillations induced by the two beams are mixed to produce a slowly varying heterodyne force driving the slow motion,

$$\dot{\phi}^{(\mathrm{sl})}+U'(\phi^{(\mathrm{sl})})=-A(t)M'(\phi^{(\mathrm{sl})})\sin\psi(t)+\Delta I(t),$$
$$A(t)=\frac{A_{\mathrm{ref}}A_{\mathrm{in}}(t)}{2\omega_0}, \tag{4}$$

where the effective potential $U(\phi)$ can be obtained[12] from Eqs. (2) and (3): in doing so, it is found that the terms $\propto A_{\mathrm{in}}, A_{\mathrm{ref}}$ drop out (to the accuracy of the corrections $\sim A_{\mathrm{in}}^2/\omega_0^2, A_{\mathrm{ref}}^2/\omega_0^2$). In the range of optical bistability the potential $U(\phi)$ is of double-well form, with minima at ϕ_1, ϕ_2 corresponding to the stable states of the system (4) in the absence of the modulation. The slowly varying force $\propto A(t)$ in (4) gives rise to a modulation of the intensity of the transmitted radiation I_T which, as we show below, can be enhanced via noise.

For small modulation amplitudes, $A_{\mathrm{in}}/\omega_0, A_{\mathrm{ref}}/\omega_0 \ll 1$, the heterodyning is fully characterized by the assumption of sinusoidal modulation, $A_{\mathrm{in}}=\mathrm{const}$, $\psi(t)=\Omega t$, and standard linear response theory may be applied to the analysis of Eqs. (1)–(4). To first order in $A=A_{\mathrm{in}}A_{\mathrm{ref}}/2\omega_0$ the intensity of the transmitted radiation is given by

$$\langle I_T(t)\rangle=\langle\overline{I_T}\rangle+\mathrm{Im}\,[\chi(\Omega)A\,\exp(-i\Omega t)],$$

where $\chi(\Omega)$ is the susceptibility. To find $\chi(\Omega)$ we notice that, for low noise intensity D, the system spends most of its time fluctuating about the stable states $n=1,2$. The susceptibility is then given by the sum of two kinds of contribution: first, from the vibrations about these states χ_n, weighted by the stationary populations of the states w_n; and secondly the term $\chi_{\mathrm{tr}}(\Omega)$ resulting from the periodic modulation of the populations by the force $A\,\exp(-i\Omega t)$,

$$\chi(\Omega)=\sum_{n=1,2} w_n\chi_n(\Omega)+\chi_{tr}(\Omega).$$

For $\Omega\ll\tau_r^{-1}$ the intrawell susceptibilities correspond to the quasistatic forcing,

$$\chi_n(\Omega)=N'(\phi_n)+M'(\phi_n)(\partial\overline{I_{Tn}}/\partial I_{\mathrm{ref}}),$$
$$\overline{I}_{Tn}=\overline{I}_{\mathrm{in}}N(\phi_n), \tag{5}$$
$$\langle\overline{I_T}\rangle=\sum_{n=1,2} w_n\overline{I_{Tn}},$$

where ϕ_n is the value of the phase ϕ in the nth stable state in the absence of modulation; it is given by the corresponding solution of Eqs. (2) and (3) with $A_{\mathrm{in}}=A_{\mathrm{ref}}=\Delta I=0$.

The term in the susceptibility due to the modulation of the populations can be evaluated along the lines[13]:

$$\chi_{tr}(\Omega)=\frac{w_1w_2}{D}[\overline{I_{T1}}-\overline{I_{T2}}][M(\phi_1)-M(\phi_2)]\times\frac{W_{12}+W_{21}}{W_{12}+W_{21}-i\Omega}. \tag{6}$$

Here, W_{nm} is the probability of the transition $n\to m$ between the stable states for $A_{\mathrm{in}}=A_{\mathrm{ref}}=0$. The quantities W_{12}, W_{21} for Brownian motion (4) were evaluated by Kramers.[14] Their dependence on the noise intensity D is of the activation type, and therefore the susceptibility $|\chi(\Omega)|$ of a bistable optical system with respect to heterodyning may increase with the increasing noise intensity.

The SNR is often taken to be inversely proportional to the product of the spectral density of the noise and a characteristic frequency resolution $\delta\omega$, and the strategy for maximising the SNR is to minimise both of these quantities. In NEOH, on the other hand, we are not greatly concerned with $\delta\omega$ (which can be minimised independently by application of the usual methods). Rather, we characterise the SNR for the heterodyning by the ratio R of the low-frequency signal in the intensity of the transmitted radiation, given by $\frac{1}{4}A^2|\chi(\Omega)|^2$, to the value of the power spectrum $Q^{(0)}(\omega)$ at the same frequency for $A=0$ which can be evaluated in a similar way to $\chi(\Omega)$. It follows immediately that for $\Omega\ll\tau_r^{-1}$, in sharp contradistinction to the standard case, R can display an exponentially sharp increase with increasing noise intensity in the range of $\overline{I}_{\mathrm{in}},\overline{I}_{\mathrm{ref}}$ for which $w_1\approx w_2$.

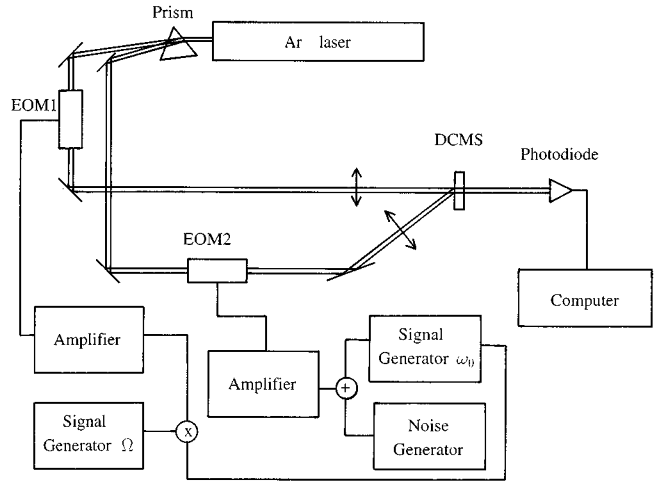

FIG. 1. Schematic diagram of the apparatus.

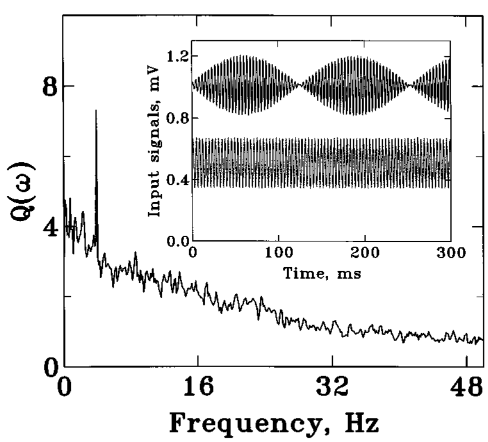

FIG. 2. Spectral density of fluctuations of the DCMS in the presence of two high-frequency fields (reference and input signals at wavelengths 488 nm and 514.5 nm respectively), for a noise intensity corresponding to that of the maximum in R (see Fig. 3). The narrow spike corresponds to the heterodyne signal. The inset shows examples of the reference signal (for ω_0 = 800 Hz, lower trace) and input signal ($\omega_0\pm\Omega$ with Ω = 3.92 Hz, upper trace), as functions of time.

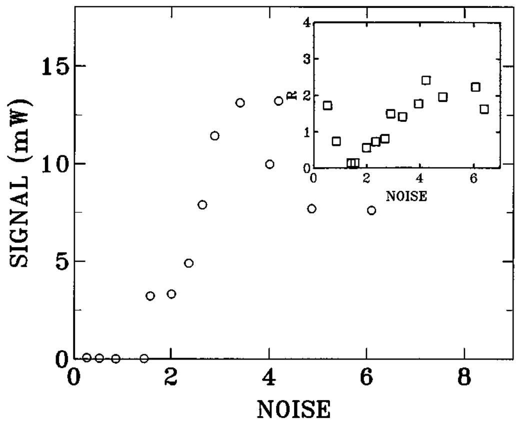

FIG. 3. Magnitude of the heterodyne signal for $\omega_0 = 2.1$ kHz, $\Omega = 3.92$ Hz as a function of noise intensity (arbitrary units). The inset shows the signal-to-noise ratio R, also as a function of noise intensity.

The experimental arrangement used to test this prediction (Fig. 1) incorporates a double-cavity membrane system (DCMS).[15] Its construction is similar to that of standard spatial-membrane light modulators,[16] except that it is operated by light instead of voltage. The first resonator consists of a ~ 1 μm membrane of semiconducting GaSe single crystal, separated from a plane dielectric mirror by a metal diaphragm ~ 500 μm in diameter. The ~ 10 μm air-filled gap between the mirror and the membrane forms a second resonator. The incident beam from an argon laser, of wavelength 514.5 nm, propagates along the normal to the mirror providing an input signal. An additional beam of wavelength 488 nm from the argon laser, inclined with respect to the DCMS axis, provides a reference signal. The intensities of the laser beams are modulated in time by two electro-optic modulators (EOM1, EOM2), to which periodic signals and noise are applied as shown. Optical bistability arises because of thermoelastic bending of the membrane caused by the main (514.5 nm) laser beam. The phase gain of the air-resonator is linear in bending and thus follows adiabatically the thermal relaxation of the film. Heating of the DCMS by the 488 nm reference signal is directly proportional to its intensity. Systems of this kind, which display optical bi- and multistability for low light intensities, are ideally suited to investigations of phenomena related to optical bistability,[17] including stochastic resonance.[6]

To seek noise-enhanced optical heterodyning, the reference signal (488 nm) was modulated periodically at a frequency ω_0 in the range 0.5–2.5 kHz (lower trace, Fig. 2 inset) and in addition by noise with a cutoff frequency of 5 kHz. The input signal (514.5 nm) was modulated at frequencies $\omega_0 \pm \Omega$, with $\Omega = 3.92$ Hz (upper trace). A heterodyne signal at frequency $\Omega = 3.92$ Hz was detected in the transmitted light intensity I_T at wavelength 514.5 nm. The measured relaxation time τ_r of the DCMS was ~ 2 ms; for most of the measurements $\omega_0 = 2.1$ kHz; and so the condition $\Omega \ll \tau_r^{-1} \ll \omega_0$ was well met. With noise-induced fluctuational transitions occurring between two stable states of the DCMS, a strong heterodyne signal (the spike in Fig. 2) appeared, superimposed on the zero-frequency Lorenzian peak in the spectral density of fluctuations of I_T with a halfwidth equal to the sum of the transition probabilities $W_{12} + W_{21}$.[12]

Strong enhancements of both the heterodyne signal (by a factor of ~ 1000) and of the signal-to-noise ratio R were observed with increasing noise intensity, as shown in Fig. 3. The dependence of R on the noise intensity (inset) is of the characteristic reversed-N shape familiar from earlier studies of SR in bistable systems[4] and consistent with the theory outlined above. The enhancement of the SNR occurs within a restricted range of noise intensities, as expected, and the ratio between the value of R at the minimum to that at the local maximum (i.e., the maximum noise-induced "amplification" of the signal-to-noise ratio) is ~ 10. A fuller discussion and a detailed quantitative comparison with the theory will be given elsewhere.

In conclusion, we have predicted, and observed experimentally, the phenomenon of noise-enhanced optical heterodyning. The advantageous features of NEOH compared to conventional heterodyning techniques are that it is intrinsically noise-protected and highly frequency selective, and that it can be performed within a single all-optical device.

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