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Physica E 22 (2004) 767–770
PHYSICA E
www.elsevier.com/locate/physe

Electron dynamics in quantum dots on helium surface

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Abstract

We study single-electron quantum dots on a helium surface created by electrodes submerged into the helium. The parameters of the dots are electrostatically controlled. We find the electron energy spectrum and identify relaxation mechanisms. The in-plane confinement significantly slows down electron relaxation. The energy relaxation is due primarily to coupling to phonons in helium. The dephasing is determined by thermally excited ripplons and by the noise from underlying electrodes. The decay rate can be further suppressed by a magnetic field normal to the helium surface. Slow relaxation in combination with control over the energy spectrum make localized electrons appealing as potential qubits of a quantum computer.

PACS: 73.21.−b; 03.67.Lx

Keywords: Electrons on helium; Quantum computers; Quantum dots; Relaxation mechanisms

The system of electrons on the surface of superfluid ^{4}He is attractive from the point of view of making a scalable quantum computer (QC) [1]. The electrons have an extremely long relaxation time and display the highest mobility known in a condensed-matter system [2]. A QC can be made by submerging a system of individually addressed micro-electrodes beneath the helium surface. The typical interelectron distance is comparatively large, $\sim$ 1 μm, which simplifies fabrication of an electrode array [3]. The electrode potential, the high barrier that prevents electrons from penetrating into the helium, and the helium image potential together create a single-electron quantum dot above each electrode. The parameters of the dot can be controlled by the electrode potential.

Here we study the energy spectrum and dissipation processes for electrons in quantum dots on helium surface. We discuss mechanisms of coupling to helium excitations, phonons and ripplons, and the dependence of the electron relaxation rate on the quantum dot parameters. We investigate the role of a magnetic field normal to the surface and of the electron–electron interaction. Decay and decoherence of the electron states result also from classical and quantum electrode noise. We relate the corresponding relaxation rates to the power spectrum of the fluctuating electric field on the electron and analyze their dependence on the parameters of the electrodes and external leads.

The geometry of a quantum dot can be understood from Fig. 1. The potential is a sum of the out-of-plane and in-plane parts. The out-of-plane potential is similar to that in the absence of the electrode. It leads to quantization of motion normal to the surface. In the absence of the field $E_\perp$ the energy levels are $E_n = -R/n^2$ $(n = 1, 2, \ldots)$, where the effective Rydberg energy $R \approx 8$ K [4]. The in-plane potential $U_\parallel(\mathbf{r}) \approx m\omega_\parallel^2 r^2/2$ $[\mathbf{r} = (x, y)]$ is parabolic, for typical parameter values.

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doi:10.1016/j.physe.2003.12.117

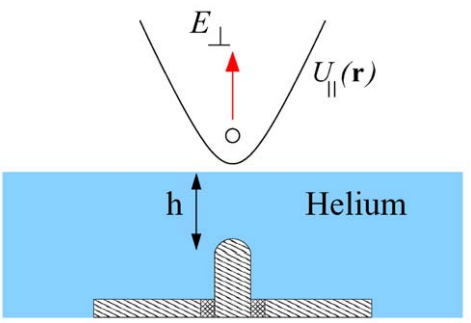

Fig. 1. Geometry of a single-electron quantum dot on helium surface. A micro-electrode is submerged by the depth $h \sim 0.5$ μm beneath the helium surface. The electron is driven by a field $E_\perp$ normal to the surface. This field is a sum of the electrode field $\mathscr{E}_\perp$ and the field of the parallel-plate capacitor (only the lower plate of the capacitor is shown). The confining in-plane potential $U_\parallel(\mathbf{r})$ $[\mathbf{r} = (x, y)]$ is determined by the electrode potential and geometry.

In the absence of a magnetic field, electron states in a dot $|n, v, m_v\rangle$ are characterized by the quantum number n of motion normal to the surface, the principal quantum number v of vibrations about an equilibrium in-plane position, and the number m_v that enumerates degenerate vibrational states.

A confined electron can serve as a qubit [1]. The working states of the qubit are $|1, 0, 0\rangle$ and $|2, 0, 0\rangle$. The energy difference between these states $E_2 - E_1$ can be Stark-shifted by the electric field from the electrode $\mathscr{E}_\perp$. The shift of 1 GHz occurs if $\mathscr{E}_\perp$ is changed by ~ 1 V/cm. The field $\mathscr{E}_\perp$ also determines the in-plane vibrational frequency $\omega_\parallel$. A simple estimate of $\mathscr{E}_\perp$ can be made by assuming that the electrodes are spheres of radius $r_{\rm el}$. In this case $\omega_\parallel = (e\mathscr{E}_\perp/mh)^{1/2}$. Typically $\omega_\parallel/2\pi \sim 20$ GHz, whereas the transition frequency $\Omega_{\rm tr} = (E_2 - E_1)/\hbar$ is 6–10 times larger.

Because of the discreteness of the electron energy spectrum in a dot, the mechanisms of electron decay and dephasing are qualitatively different from those studied for a 2D electron system on helium in the absence of in-plane confinement [5]. In particular, the major known scattering mechanism, quasi-elastic scattering by ripplons, does not work. Decay of the excited state $|2, 0, 0\rangle$ is most likely to occur via a ripplon- or phonon-induced transition to the closest in energy excited vibrational state of the electron $|1, v_{\rm c}, m_v\rangle$, with $v_{\rm c} = {\rm int}[(E_2 - E_1)/\hbar\omega_\parallel]$. The energy transfer in the transition is $\delta E = E_2 - E_1 - v_{\rm c}\hbar\omega_\parallel \sim \hbar\omega_\parallel$. It largely exceeds the energy of ripplons with wave numbers $q \lesssim 1/a_\parallel$, where $a_\parallel = (\hbar/m\omega_\parallel)^{1/2}$ is the in-plane electron localization length in the dot. This makes one-ripplon decay exponentially improbable and strongly reduces the decay rate compared to the case of electrons that are free to move along the helium surface.

An electron in a dot can decay by emitting two ripplons that will propagate in opposite directions with nearly same wave numbers $q_{1,2}$, which are determined by the condition that the ripplon frequency is $\omega_{\rm r}(q_{1,2}) \approx \delta E/2\hbar$ $[|\mathbf{q}_1 + \mathbf{q}_2| \lesssim 1/a_\parallel \ll q_{1,2}]$. Alternatively, and even with higher probability, decay may occur through an electron transition accompanied by emission of a phonon. The appropriate phonons propagate nearly normal to the helium surface: their wave vectors make an angle with the normal to the surface $\sim (mv_{\rm s}^2/\hbar\omega_\parallel)^{1/2} \ll 1$, where $v_{\rm s}$ is the sound velocity in helium. We propose two mechanisms of coupling to phonons [6]. One is phonon-induced deformation of the helium surface. The second is phonon-induced modulation of the dielectric constant of helium and thus of the electrostatic energy of the electron above helium.

Relatively simple expressions for the electron decay rate for different coupling mechanisms are obtained by approximating the electron potential at the helium surface by a sharp infinitely high potential wall. Such an approximation applies only if the wavelength of helium vibrations is much bigger than the width $\Delta_{\rm d}$ of the diffuse surface layer, which is of the order of a few angstroms [7]. When we estimate the decay rate numerically, we make an assumption that helium vibrations are essentially decoupled from the electrons when their wave numbers exceed $\Delta_{\rm d}^{-1}$. Then, for typical in-plane electron frequencies $\omega_\parallel/2\pi \sim 20$ GHz, both the two-ripplon and phonon decay rates are determined primarily by electron transitions with the smallest energy transfer $\delta E \sim \hbar\omega_\parallel$. Transitions over several electron vibrational levels, with energy transfer $n\hbar\omega_\parallel$ with $n \gg 1$, can be disregarded. This gives the decay rate $\lesssim 10^4$ s^{-1}, which is presumably an overestimate.

Electron dephasing due to coupling to excitations in helium comes primarily from quasi-elastic scattering of ripplons off the electron. Scattering amplitudes are different in different electron states, and therefore scattering leads to diffusion of the phase difference between the wave functions $|2, 0, 0\rangle$ and $|1, 0, 0\rangle$. The typical wave numbers q_ϕ of the ripplons that contribute to dephasing are determined by the condition that the ripplon frequency $\omega_{\rm r}(q_\phi) \lesssim k_{\rm B}T/\hbar$. They are much smaller than $1/\Delta_{\rm d}$, and the coupling is well

described by the approximation of a sharp helium surface. For different mechanisms of coupling to ripplons the dephasing rate $\Gamma^{(\phi)}$ displays same temperature dependence $\Gamma^{(\phi)} \propto T^3$, which is much slower than the standard T^7 law for defects in solids. Numerically, $\Gamma^{(\phi)} \lesssim 10^2\ \mathrm{s}^{-1}$ for $T = 10$ mK.

The electron decay rate may be further reduced by applying a magnetic field normal to the helium surface. As we will show, this is equivalent to further increasing $\omega_{\|}$ and should require helium vibrations with even shorter wavelengths in order to meet the condition of energy conservation in decay. The theory will also describe the case of many quantum dots. In this case, the energy spectrum of in-plane electron excitations consists of plasmon bands and is continuous. Still, as we show, the relaxation rate remains strongly suppressed compared to the case of unconfined electrons.

The in-plane electron coordinate $\mathbf{r}_n$ in an nth dot can be expanded in the creation and annihilation operators of the in-plane vibrational modes of the electrons a_{kj}, a_{kj}^{+},

$$\mathbf{r}_n = \mathbf{R}_n + \sum_{kj} [\mathbf{A}_{kj}^{(n)} a_{kj} + \mathrm{h.c}]. \tag{1}$$

Here, $\mathbf{R}_n$ is the equilibrium in-plane position, and $j = 1, 2$ enumerates vibrational modes. The quantum number k can be set equal to zero in the case of one dot, whereas for a periodic array of dots it becomes a plasmon wave vector $\mathbf{k}$.

For one dot ($n = 1$) in a strong magnetic field $\mathbf{B}$ antiparallel to $\hat{\boldsymbol{z}}$

$$\omega_{01} \approx \omega_c = |eB|/mc, \quad \omega_{02} \approx \Omega_0 \equiv \omega_{\|}^2/\omega_c,$$

$$\mathbf{A}_1 \approx \mathrm{i}\mathbf{A}_2 \approx 2^{-1/2} l(\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}}) \quad [\mathbf{A}_j \equiv \mathbf{A}_{0j}^{(1)}], \tag{2}$$

where $l = (\hbar/m\omega_c)^{1/2}$ is the magnetic length, and we assumed that $\omega_{\|}$ is small compared to the cyclotron frequency ω_c. For a multi-dot system, the vibrational frequencies form two bands. The bandwidths are $\lesssim \omega_p^2/\omega_c$, where $\omega_p = (2\pi e^2/md^3)^{1/2}$ is the typical plasma frequency ($\omega_p \ll \omega_c$) and d is the interelectron distance. They are further reduced if $\omega_p \ll \omega_{\|}$. The minimal frequency of the upper band is $\approx \omega_c$, whereas that of the lower band is $\sim \Omega_0$.

The Hamiltonian that describes $|2\rangle \to |1\rangle$ transitions induced by excitations in helium has the form

$$H_i^{(\mathrm{d})} = \sum_n |2\rangle_{n\,n}\langle 1| \sum_{\mathbf{q}} \hat{V}_{\mathbf{q}} \mathrm{e}^{\mathrm{i}\mathbf{q}\mathbf{r}_n} + \mathrm{h.c.} \tag{3}$$

Here, $|1\rangle_n$ and $|2\rangle_n$ are the states of an nth electron normal to the surface, and $\hat{V}_{\mathbf{q}}$ is the operator that depends on the coordinates of helium vibrations, i.e., phonons and ripplons. The wavelengths of the vibrations involved in electron scattering are much smaller than the interelectron distance. Therefore each electron has its "own" thermal bath of helium excitations. In the Born approximation, the decay rate of the state $|2\rangle_n$ for an nth electron is

$$\Gamma_n^{(\mathrm{d})} = \hbar^{-2}\mathrm{Re}\int_0^{\infty} \mathrm{d}t \mathrm{e}^{\mathrm{i}\Omega_{\mathrm{tr}}t} \sum_{\mathbf{q}} \tilde{\mathscr{S}}_n(\mathbf{q}, t) \times \langle \hat{V}_{\mathbf{q}}(t)\hat{V}_{-\mathbf{q}}(0)\rangle,$$

$$\tilde{\mathscr{S}}_n(\mathbf{q}, t) = \langle \mathrm{e}^{\mathrm{i}\mathbf{q}\mathbf{r}_n(t)} \mathrm{e}^{-\mathrm{i}\mathbf{q}\mathbf{r}_n(0)}\rangle. \tag{4}$$

Here, the averaging $\langle\cdot\rangle$ is performed assuming that the electron and helium vibrations are uncoupled. We have also disregarded the difference between the transition frequencies $\Omega_{\mathrm{tr}} = (E_2 - E_1)/\hbar$ for different electrons.

From Eq. (1), for $k_B T \ll \hbar\Omega_0$ the structure factor $\tilde{\mathscr{S}}_n(\mathbf{q}, t)$ (4) has the form

$$\tilde{\mathscr{S}}_n(\mathbf{q}, t) = \exp[-q^2 W_n(t)/2],$$

$$W_n(t) = \sum_{kj} |\mathbf{A}_{kj}^{(n)}|^2 [1 - \exp(-\mathrm{i}\omega_{kj}t)]. \tag{5}$$

The effective Debye–Waller factor $W_n(t)$ is independent of the electron number n if the coefficients $\mathbf{A}_{kj}^{(n)}$ for different electrons differ only by a phase factor, as in the case of a periodic set of dots.

In what follows we will assume that the level spacing $E_2 - E_1$ is of the same order as the distance between the Landau levels $\hbar\omega_c$ and that the energy deficit $\delta\tilde{E} = E_2 - E_1 - \tilde{\nu}_c\hbar\omega_c$ largely exceeds $\hbar\Omega_0$ [here, $\tilde{\nu}_c = \mathrm{int}(E_2 - E_1)/\hbar\omega_c$]. Then many vibrations of the lower vibrational branch $j = 2$ are excited in a transition. This means that the structure factor $\tilde{\mathscr{S}}_n(\mathbf{q}, t)$ can be evaluated assuming that $\Omega_0 t \ll 1$. In doing this we will take into account the relations $\sum_k |\mathbf{A}_{k1}^{(n)}|^2 \approx l^2$ and

$$\sum_k |\mathbf{A}_{k2}^{(n)}|^2 \omega_{k2} \approx l^2\Omega_n, \Omega_n = [\partial^2 U_{\|}/\partial \mathbf{r}_n^2]/2m\omega_c, \tag{6}$$

where $U_{\|}$ is the overall in-plane potential of an nth electron that includes the Coulomb energy of the electron-electron interaction, and the Laplacian is calculated at the equilibrium position. It is seen from Eq.

(2) that Eq. (6) applies in the case of one dot. A proof for a multi-dot system will be discussed elsewhere.

From Eq. (6) we obtain

$$\tilde{\mathscr{S}}_n(\mathbf{q},t) \approx \exp[-(q^2l^2/2)(1+\mathrm{i}\Omega_n t)] \times \sum_v \frac{1}{v!}(q^2l^2/2)^v \,\mathrm{e}^{-\mathrm{i}v\omega_c t}. \quad (7)$$

This equation shows that, in an electron $|2\rangle \to |1\rangle$ transition, the energy transferred to the lower-branch vibrational modes is $\sim q^2l^2\Omega_n\hbar$. The typical values of the factor q^2l^2 are $\lesssim 1$, otherwise the transition probability becomes exponentially small. This means that, even though the energy spectrum of a set of dots is band-like, the typical energy that has to be transferred to helium excitations is $\sim \delta\tilde{E} \sim \hbar\omega_c$. For $\omega_\parallel \ll \omega_c$ this is a much bigger energy than in the absence of a magnetic field. Therefore we expect that a magnetic field can significantly reduce the decay rate, because decay requires helium excitations with wavelengths smaller than the width of the surface diffuse layer.

The occurrence of a mode with frequency $\Omega_0 \ll \omega_\parallel$ in a magnetic field leads to an increase of the ripplon-induced dephasing rate. This can be seen in the higher order in the electron–ripplon coupling where account is taken of virtual ripplon-induced transitions between the electron vibrational levels, cf. Refs. [1,6]. The amplitude of such transitions increases with the decreasing Ω_0, i.e., with the increasing magnetic field. The effect imposes a limitation on the field magnitude. One concludes that there exists an optimal range of magnetic fields where both decay and dephasing rates are small and of the same order of magnitude.

An important source of dephasing of electron states in a single-electron dot is noise from the underlying electrode, see Fig. 1. We will discuss it for one dot, as the role of inter-dot interaction is minor in this case. Coupling to the electrode noise is dipolar, with the Hamiltonian

$$H_{\mathrm{dip}} = e\delta\hat{\mathscr{E}}_\perp z, \quad (8)$$

where $\delta\hat{\mathscr{E}}_\perp$ is the normal to the surface component of the field from quantum or classical charge density fluctuations in the electrode.

The rates $\Gamma_{\mathrm{el}}^{(\phi)}$ and $\Gamma_{\mathrm{el}}^{(\mathrm{d})}$ of the electron dephasing and decay can be expressed [6] in terms of the field correlation function

$$Q(\omega) = \int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t}\langle \delta\hat{\mathscr{E}}_\perp(t)\delta\hat{\mathscr{E}}_\perp(0)\rangle \quad (9)$$

as

$$\Gamma_{\mathrm{el}}^{(\phi)} = e^2(z_{22}-z_{11})^2 \mathrm{Re}\, Q(0)/\hbar^2,$$

$$\Gamma_{\mathrm{el}}^{(\mathrm{d})} = e^2|z_{12}|^2 \mathrm{Re}\, Q(\Omega_{\mathrm{tr}})/\hbar^2. \quad (10)$$

Here, $z_{ij} = \langle i|z|j\rangle$, with $i,j = 1,2$.

A major contribution to the dephasing rate comes from Johnson's noise in the external leads. A simple estimate can be made by assuming that the confining electrode is a sphere of radius r_{el}. It gives

$$\Gamma_{\mathrm{el}}^{(\phi)} = 2k_B T_{\mathrm{ext}} \mathscr{R}_{\mathrm{ext}} e^2 (z_{22}-z_{11})^2 r_{\mathrm{el}}^2/\hbar^2 h^4, \quad (11)$$

where T_{ext} and $\mathscr{R}_{\mathrm{ext}}$ are the lead temperature and resistance. For $\mathscr{R}_{\mathrm{ext}} = 25\ \Omega$, $T_{\mathrm{ext}} = 1$ K, $r_{\mathrm{el}} = 0.1$ μm, and $h = 0.5$ μm we obtain $\Gamma_{\mathrm{el}}^{(\phi)} \approx 1 \times 10^4\ \mathrm{s}^{-1}$. Eq. (11) suggests how to reduce the rate $\Gamma_{\mathrm{el}}^{(\phi)}$. The decay rate $\Gamma_{\mathrm{el}}^{(\mathrm{d})}$ is much less than the phonon-induced decay rate.

Eq. (10) allows one also to estimate the effect of noise from reorienting defects in the electrode. For electrodes submerged into helium this noise should be weaker than in semiconductor-based systems proposed for quantum computing, in particular because it scales with the distance to the electrode as h^{-4}, cf. Eq. (11).

In this paper we have calculated the relaxation rate for electrons in quantum dots on the helium surface. We proposed new relaxation mechanisms and found the dependence of the relaxation rate on the potential of the confining electrodes and the magnetic field. It follows from the results that the ratio of the relaxation rate to the clock frequency of the QC based on electrons on helium, which is determined by the electron–electron interaction, can be as small as 10^{-4}–10^{-5}, for typical interdot spacing $\lesssim 1$ μm.

References

[1] P.M. Platzman, M.I. Dykman, Science 284 (1999) 1967; M.I. Dykman, P.M. Platzman, Quantum Inf. Comput. 1 (2001) 102.
[2] K. Shirahama, S. Ito, H. Suto, K. Kono, J. Low Temp. Phys. 101 (1995) 439.
[3] J.M. Goodkind, S. Pilla, Quantum Inf. Comput. 1 (2001) 108.
[4] E.Y. Andrei (Ed.), Two Dimensional Electron Systems on Helium and Other Cryogenic Substrates, Kluwer, Dordrecht, 1997.
[5] T. Ando, J. Phys. Soc. Jpn. 44 (1978) 765.
[6] M.I. Dykman, P.M. Platzman, P. Seddighrad, Phys. Rev. B 67 (2003) 155402.
[7] See K. Penanen et al., Phys. Rev. B 62 (2000) 9621 and references therein.