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Ripplon-induced tunneling transverse to the magnetic field

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We analyze single-electron tunneling from the bound state above the surface of liquid helium. For strong enough magnetic fields parallel to the surface, the tunneling must be accompanied by ripplon scattering. The effective width of the tunneling barrier strongly and nonmonotonically depends on the momentum transferred to ripplons. The escape rate is affected by Landau quantization of the states behind the barrier. The results obtained here also apply to electrons on other cryogenic substrates. [S0163-1829(98)50444-4]

Magnetic field B parallel to the two-dimensional (2D) electron layer can strongly suppress the probability of tunneling from the layer. This happens because the field effectively adds to the potential barrier $U(z)$ an extra term

$$\delta U(z;p_H)=\frac{1}{2}m\omega_c^2z^2+\omega_c p_H z, \tag{1}$$

where z is the tunneling direction, $\omega_c=eB/m$ is the cyclotron frequency, and p_H is the component of the electron momentum in the Hall direction $\mathbf{B}\times\hat{\mathbf{z}}$. Suppression of tunneling has been observed[1,2] and investigated experimentally[2] and theoretically[3] for tunneling between 2D electron layers in semiconductor heterostructures, and also for tunneling out of a 2D electron layer on helium surface.[4] However, the observed decrease of the tunneling current with the increasing magnetic field was smaller, and for electrons on helium *much* smaller than what follows from calculations that omitted collisions.

In the present paper we investigate single-electron tunneling from 2D layers on helium. We show that, for strong magnetic fields, the tunneling is accompanied by emission or absorption of ripplons, capillary waves on helium surface, in which a part of the electron momentum in the Hall direction is transferred to a ripplon. As a result the magnetic barrier (1) is reduced, and therefore the tunneling probability increases exponentially.[5]

A distinctive feature of electrons on helium, which makes the problem of tunneling particularly interesting and different from what has been discussed for semiconductor heterostructures, is that the tunneling barrier $U(z)$ is *smooth* (linear) on its ''external'' side, see Fig. 1. This barrier is created by the image force and by the applied electric field $\mathcal{E}_\perp$ which pulls the electrons *away* from the surface. In the absence of the magnetic field

$$U(z)=-\frac{\Lambda}{z}-e\mathcal{E}_\perp z. \tag{2}$$

Here, $\Lambda=e^2(\epsilon-1)/8$ is the image-force parameter, and we assume that helium occupies the half-space $z\leqslant 0$.

Helium is essentially impenetrable for low-energy electrons, and for weak electric fields the electrons reside in the ground state of the potential well formed by the infinite wall at $z=0$ and the image-force potential. The wave function $\psi_g(z)$ and the energy E_g of this state are given by the expressions

$$\psi_g(z)=2\gamma^{3/2}z\,\exp(-\gamma z),\quad E_g=-\hbar^2\gamma^2/2m, \tag{3}$$

where $\gamma=m\Lambda/\hbar^2$ is the reciprocal localization length.

The width of the tunneling barrier for weak electric field is

$$L=|E_g|/e\mathcal{E}_\perp, \tag{4}$$

and we have $\gamma L\gg 1$ (this condition is necessary for the tunneling rate to be small).

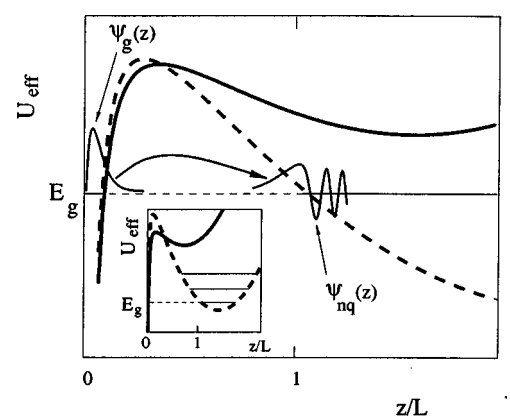

FIG. 1. The effective potential energy $U(z)+\delta U(z,\hbar q)$ in the magnetic field for zero (solid line) and close to optimal (dashed line) electron momentum $\hbar q$ in the direction $-\mathbf{B}\times\hat{\mathbf{z}}$. Transitions between the state localized on the helium surface and the states behind the barrier are induced by ripplons. Inset: Landau quantization of the states behind the barrier. For large B, electrons make transitions to the lowest Landau level.

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Because of the smoothness of $U(z)$ for $z\sim L$, a sufficiently strong magnetic field B parallel to the surface eliminates the states behind the overall barrier $U(z)+\delta U(z;p_H)$ with the energy E_g of the ground intrawell state and zero Hall momentum $p_H=0$.[4] This happens for $\alpha>1$, where

$$\alpha=4\delta U(L;0)/|E_g|\equiv 2m\omega_c^2 L/e\mathcal{E}_\perp . \tag{5}$$

The momentum transfer from ripplons, which is analyzed in the present paper, can restore the tunneling. With account taken of this transfer, the effective width of the tunneling barrier is given by the condition $U(z)+\delta U(z;p_H)=E_g$. It strongly and nonmonotonically depends on the transferred momentum p_H. Therefore the probability density of ripplon-induced transitions sharply peaks at an optimal ripplon momentum. In calculating this momentum and the transition rate it is necessary to allow for the Landau quantization of the states behind the barrier (see Fig. 1).

The mechanism of ripplon-induced tunneling is different from that of defect-induced tunneling in semiconductors.[5] Vibrations of the helium surface result in a coordinate-dependent perturbation of the boundary conditions for the electron wave function. The interaction Hamiltonian H_i is obtained by changing to variables $\mathbf{r}\to\mathbf{r}, z\to z-\xi(\mathbf{r})$, where $\xi(\mathbf{r})$ is the ripplon-induced displacement of the helium surface [cf. Refs. 6 and 7; $\mathbf{r}=(x,y)$ is the in-plane electron coordinate]. To first order in $\xi(\mathbf{r})$,

$$H_i=\sum_{\mathbf{q}} \xi_{\mathbf{q}} e^{i\mathbf{qr}}\hat{V}_{\mathbf{q}}(\hat{\mathbf{P}},\hat{p}_z,z), \quad \xi(\mathbf{r})=\sum_{\mathbf{q}} \xi_{\mathbf{q}} e^{i\mathbf{qr}}, \tag{6}$$

with

$$\hat{V}_{\mathbf{q}}(\hat{\mathbf{P}},\hat{p}_z,z)=-\frac{i}{m}(\mathbf{q}\cdot\hat{\mathbf{P}})\hat{p}_z-\frac{i\hbar}{2m}q^2\hat{p}_z+\omega_c\hat{P}_x-e\mathcal{E}_\perp + \Lambda K_q(z).$$

Here, $\hat{\mathbf{P}}=(-i\hbar\nabla_x+m\omega_c z,-i\hbar\nabla_y)$ is the 2D electron momentum, and we chose $\mathbf{B}$ to be pointing in the y direction (respectively, the Hall momentum $\mathbf{p}_H$ is pointing in the x direction).

The first two terms in the operator $\hat{V}_{\mathbf{q}}$ describe a *kinematic* interaction with ripplons which is due to the curvature of the surface on which the electron wave function is set equal to 0. The polarization interaction $K_{\mathbf{q}}(z)$ is given in Ref. 7. The kinematic interaction turns out to be more important for the tunneling than the change of the potential energy due to surface displacement.

Even for the lowest temperatures used in the experiments on electrons on helium (see Ref. 8), the surface displacement $\xi(\mathbf{r})$ is classical and quasistatic for typical wavelengths $\lesssim 10^{-5}$ cm. For many purposes it is an ideal zero-mean Gaussian random field, with the correlator $\langle|\xi_{\mathbf{q}}|^2\rangle = k_BT/S\sigma q^2$, where σ is the surface tension and S is the area of the system.

The electron-ripplon interaction (6) gives rise to mixing of the intrawell state (3) and the external states on the opposite side of the barrier, as shown in Fig. 1. For $\gamma L\gg 1$, the wave functions $\psi_{n\mathbf{q}}(z)$ of the external states are to a good approximation eigenfunctions of an electron in crossed electric and magnetic fields $\mathcal{E}_\perp$ and B, with energies

$$E_{n\mathbf{q}}=\hbar\omega_c\left(n+\frac{1}{2}\right)+\frac{\hbar^2 q_y^2}{2m}+\frac{e\mathcal{E}_\perp}{m\omega_c}\left(\hbar q_x-\frac{e\mathcal{E}_\perp}{2\omega_c}\right). \tag{7}$$

To the lowest order in the coupling to ripplons, the rate of ripplon-induced transitions W_r is given by

$$W_r=\frac{2\pi}{\hbar}\sum_{n,\mathbf{q}}\langle|\xi_{\mathbf{q}}|^2\rangle|(\psi_g|\hat{V}_{\mathbf{q}}|\psi_{n\mathbf{q}})|^2\delta(E_g-E_{n\mathbf{q}}). \tag{8}$$

Once the electron tunnels into the state $\psi_{n\mathbf{q}}$, it drifts in crossed B and $\mathcal{E}_\perp$ fields parallel to the helium surface. However, in a relatively short time it is scattered by the electrons localized on the surface and moves away from the surface. Therefore the bottleneck for escape is reaching an external state, and the overall escape rate is given by W_r.[9]

The matrix element in Eq. (8) involves the overlap of the two wave functions, ψ_g and $\psi_{n\mathbf{q}}$. These functions decay exponentially from opposite ends of the barrier, see Fig. 1. Therefore we only need to know their tails. The tail of $\psi_g(z)$ for $z\gg 1/\gamma$ is given by the WKB approximation $\psi_g(z)\propto\exp[-S_1(z;\mathbf{0})/\hbar]$, where

$$S_1(z;\mathbf{q})=\int_{2/\gamma}^{z} dz'|p_z(z';\mathbf{q})|, \tag{9}$$

and p_z is the momentum in the z direction as a function of z and the in-plane momentum $\hbar\mathbf{q}$,

$$p_z(z;\mathbf{q})=(2m[E_g-U(z)]-(m\omega_c z+\hbar q_x)^2-\hbar^2 q_y^2)^{1/2}$$

(the lower limit in the integral for S_1 is set at the turning point; it affects only the prefactor in ψ_g).

Similarly, $\psi_{n\mathbf{q}}(z,\mathbf{r})\propto\exp[i\mathbf{qr}-S_2(z,\mathbf{q})/\hbar]$ with

$$S_2(z;\mathbf{q})=\int_{z}^{z_t} dz'|p_z(z';\mathbf{q})|, \tag{10}$$

where $z_t\equiv z_t(\mathbf{q})$ is the turning point on the external side of the barrier defined by the condition $p_z(z_t;\mathbf{q})=0$.

The matrix elements in Eq. (8) are evaluated for the wave numbers $\mathbf{q}_n$ which satisfy the condition $E_{n\mathbf{q}_n}=E_g$. The overlap of the wave functions is $\psi_g(z)\psi_{n\mathbf{q}_n}(z,\mathbf{r})\propto\exp[-S(z;\mathbf{q}_n)/\hbar]$, where

$$S(z;\mathbf{q}_n)=S_1(z;\mathbf{0})+S_2(z;\mathbf{q}_n). \tag{11}$$

The function S (11) has a minimum, so that $\exp[-S(z;\mathbf{q}_n)/\hbar]$ has a sharp peak for $z=\overline{z}(\mathbf{q}_n)$, where $\overline{z}$ is defined by $|p_z(\overline{z};\mathbf{0})|=|p_z(\overline{z};\mathbf{q}_n)|$. The matrix elements (i.e., the integrals over z) in Eq. (8) can be therefore calculated by the steepest descent method and are $\propto\exp[-S(\overline{z};\mathbf{q}_n)/\hbar]$.

In evaluating $S(\overline{z};\mathbf{q}_n)$ it is convenient to scale the electron momentum $\hbar\mathbf{q}$ by $2m\mathcal{E}_\perp/B$ and introduce the respective dimensionless momentum component Q transverse to the magnetic field and the dimensionless kinetic energy $Q-Q_n=(B^2/2m\mathcal{E}_\perp^2)\hbar^2 q_y^2/2m$ of the motion along the magnetic field in the nth Landau band (7),

$$Q=-(\hbar B/2m\mathcal{E}_\perp)q_x, \quad Q_n=(2n+1)/4G+Q_b,$$

$$Q_b=(\alpha-1)/4, \quad G=2\alpha^{-3/2}\gamma L. \tag{12}$$

Here, Q_n is the minimal value of $Q \propto q_x$ in the nth band. It is given by the condition $E_{nq_x}=E_g$. The quantity Q_b is the value of Q for which the bottom of the potential well $-e\mathcal{E}_\perp z+\delta U(z;\hbar q_x)$ behind the barrier in Fig. 1 lies at the energy E_g of the localized state on the helium surface.

For $\gamma L \gg 1$, the expression for S (11) can be simplified by noticing that, for characteristic $z \gg 1/\gamma$, the image-force term $-\Lambda/z$ in the electron energy (2) is small. Then $S(\overline{z};\mathbf{q}_n)=\hbar G R(Q,Q_n)$, where

$$R(Q,Q_n)=Q[4Q_b+(Q-Q_n/Q)^2]^{1/2}-2Q_b\ln[(\alpha^{1/2}-1)/t_-]+2(Q_n-Q_b)\ln[2(Q_n-Q_b)^{1/2}/t_+]+\alpha^{1/2}/2,$$

$$t_\pm=Q\pm Q_n/Q+[(Q-Q_n/Q)^2+4Q_b]^{1/2}. \qquad (13)$$

The major contribution to the escape rate W_r (8) comes from the range of q_x,n, where the function $R(Q,Q_n)$ is close to its minimum. We will first assume that one can change from the sum over n to the integral over Q_n. The integrals over Q and Q_n in Eq. (8) *may not* be independently evaluated by the steepest descent method, because of the constraint $Q-Q_n \propto \hbar^2 q_y^2/2m \geq 0$. For a given Q, $R(Q,Q_n)$ is *monotonically decreasing* with the increasing Q_n (i.e., with the decreasing q_y^2) in the allowed range $Q_n \leq Q$. Therefore R is minimal for $Q=Q_n$. The position of the minimum $\overline{Q}$ is given by the equation,

$$dR(\overline{Q},\overline{Q})/d\overline{Q}=0. \qquad (14)$$

Equations (13) and (14) result in a simple algebraic equation for $\overline{Q}$. By solving it one finds that $\overline{Q}$ varies from 0.2α for $\alpha \ll 1$ to $\alpha/4$ for $\alpha \gg 1$. This shows that the optimal momentum transferred by ripplons $\hbar|\overline{q_x}|$ and the extreme point $\overline{z}=2L\overline{Q}/\alpha$ where the ripplon scattering ''occurs'' remain close to half the characteristic Hall momentum eBL and the midpoint $L/2$ of the $B=0$ barrier, respectively. This justifies the neglect of image forces in Eq. (13).

In calculating the integral over q_x in Eq. (8) for Q,Q_n close to $\overline{Q}$ one should keep only the linear term in the expansion of R in $Q-Q_n$. It follows from Eqs. (11)–(13) that the characteristic $Q-Q_n$ which contribute to the integral over $\mathbf{q}$ are $\propto(\gamma L)^{-1}$. The characteristic $|Q_n-\overline{Q}| \propto (\gamma L)^{-1/2}$ are much larger where the sum over n in (8) can be replaced by an integral. Evaluating the integral over Q_n by the steepest descent method, we obtain

$$W_r=C\ \exp[-\gamma L\mathcal{R}(\alpha)],\quad \mathcal{R}(\alpha)=4\alpha^{-3/2}R(\overline{Q},\overline{Q}), \qquad (15)$$

$$\mathcal{R}(\alpha)\approx\frac{1}{4}\alpha^{1/2}\left[1+\frac{2}{\alpha}\left(1+2\ln\frac{\alpha}{4}\right)\right]\quad \text{for}\quad \alpha\gg 1.$$

The function $\mathcal{R}(\alpha)$ is shown in Fig. 2. It gives the *exponent* in the rate of ripplon-induced escape as a function of the magnetic field ($\alpha \propto B^2$) and monotonically increases with α. For small fields, $\alpha \ll 1$, $\mathcal{R}\approx\mathcal{R}(0)=4/3$, and $\gamma L\mathcal{R}$ coincides with the exponent of the rate of tunneling without ripplons for $B=0$.

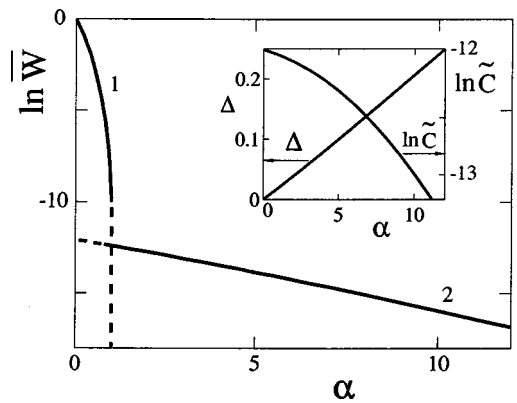

FIG. 2. Comparison of the tunneling rate (line 1) and the rate W_r (15), (16) of ripplon-induced escape (line 2) for $\gamma L=15$, $T=0.04$ K ($\overline{W}$ is the escape rate scaled by the tunneling rate for $B=0$). Inset: the effective action $\Delta=\mathcal{R}(\alpha)-\mathcal{R}(0)$ and the scaled prefactor $\tilde{C}$ in W_r as a function of the scaled magnetic field $\alpha^{1/2}=\hbar\gamma\omega_c/e\mathcal{E}_\perp$.

The prefactor C in the escape rate can be conveniently written in the form of a product of the prefactor in the tunneling rate for $B=0$ and a dimensionless parameter $\tilde{C}$,

$$C=[\hbar\gamma^2/m\ \exp(2)]\tilde{C},$$

$$\tilde{C}=2kT\gamma^2\overline{Q}(\overline{Q}^2-2\overline{Q}+\alpha)^{1/4}/\alpha\sigma(R'')^{1/2}, \qquad (16)$$

where $R''=d^2R(\overline{Q},\overline{Q})/d\overline{Q}^2$. We note that, in the range of interest where $\alpha \gtrsim 1$, the prefactor is determined by the kinematic terms in Eq. (6). The corrections from other terms are $\sim 1/\gamma L$.

For $\alpha \lesssim 1$, the electron is scattered by ripplons into highly excited Landau bands behind the barrier, with the band number $n \sim G(\overline{Q}-Q_b)$. (We note that the steepest descent method applies provided $\alpha \gg 1/\gamma L$.)

With the increasing α the characteristic n decreases [as $\exp(-\alpha/2)$ for large α], and eventually becomes small. In this case the Landau quantization behind the barrier should become substantial (see Fig. 1). Since the energy of the final state $E_{n\mathbf{q}}$ is equal to E_g, one might expect that the transition rate would display Shubnikov–de Haas-type oscillations with B each time the bottom of a Landau band crosses E_g. However, this does not happen. The physical reason is that the positions of the Landau bands (7) depend on the transferred momentum $\hbar q_x$, which is adjusted so as to maximize the escape rate.

For large α, escape is accompanied by the momentum transfer $\sim Q_b$ in dimensionless units. The function R is singular at Q_b. For $Q-Q_b$, $Q_n-Q_b \ll 1$ and $\alpha \gg 1$

$$R(Q,Q_n)\approx R(Q_b,Q_b)+(\alpha+3)(Q-Q_b)/2-(Q_n-Q_b)\ln[e\alpha^2/16(Q_n-Q_b)]. \qquad (17)$$

It follows from Eqs. (14) and (17) that the characteristic n is determined by the parameter

$$\beta=\alpha^{1/2}\gamma L\ \exp(-\alpha/2),\quad \gamma L\gg\alpha^{1/2}\gg 1, \qquad (18)$$

which depends on α *exponentially strongly*. For $\beta \gg 1$, even though α is large, the escape rate is dominated by the transitions to highly excited Landau bands, and W_r is then given by Eq. (15).

For $\beta \sim 1$, one cannot replace the sum over n in Eq. (8) by an integral. The escape rate is then a sum of ''partial'' escape rates to different Landau bands behind the barrier,

$$W_r = C[R''/4\pi G]^{1/2} \sum_n \exp[-2GR(Q_n, Q_n)]. \quad (19)$$

When $\beta \ll 1$, the major contribution to the escape rate W_r (19) comes from the transitions to the lowest Landau band $n=0$, and only the term with $n=0$ should be retained in Eq. (19).

The prefactor in (19) has been obtained in the WKB approximation for the functions $\psi_{n\mathbf{q}}$. For small n, one should allow for an extra n-dependent factor in $\psi_{n\mathbf{q}}$, which is numerically close to 1. In particular, for $n=0$ it is equal to $(\pi/e)^{1/4}$. Interestingly, with account taken of this factor (squared), Eq. (19) for $\beta \ll 1$ *goes over* into Eq. (15) of the steepest descent method. This behavior resembles some results of the instanton theory [cf. Ref. 10]. Overall, in the whole range of β, the sum (19) is well approximated by Eq. (15).

Numerical results on the ripplon-induced escape rate are shown in Fig. 2. The electron-ripplon interaction is very weak, and therefore the crossover from the tunneling without ripplons to the ripplon-induced one occurs for α very close to 1. The dependence of the exponent $\mathcal{R}$ on the magnetic field is comparatively weak for $\alpha \sim 1$, in contrast to the extremely steep dependence of the tunneling rate without ripplons, which is a signature of the mechanism we have discussed. The specific dependence of the exponent $\gamma L \mathcal{R} \propto B/\mathcal{E}_\perp^2$ on the fields $B, \mathcal{E}_\perp$ for $\alpha \gg 1$ may be used to identify the mechanism under consideration experimentally. We note that the existing data[4] refer to the range $\alpha < 1$ where ripplons essentially do not affect the escape rate.

It follows from the above analysis that ripplon scattering eliminates magnetic-field-induced localization of electrons in the smooth potential barrier on helium surface. An interesting feature of tunneling through a smooth barrier is that, for strong magnetic fields, quantization of electron states behind the barrier becomes substantial. The results apply directly to electrons localized on other cryogenic surfaces, including ^{3}He and liquid and solid neon and hydrogen. We note that, for electrons on solid substrates, the scattering will be due to surface defects, in which case the scattering rate is higher, and the crossover to scattering-induced tunneling will occur for smaller α.

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