

Quantum Measurements with Dynamically Bistable Systems

M. I. Dykman

Abstract Periodically modulated nonlinear oscillators often display bistability of forced vibrations. This bistability can be used for new types of quantum measurements. They are based on switching between coexisting vibrational states. Since switching is accompanied by a large change of the amplitude and phase of forced vibrations, the measurements are highly sensitive. Quantum and classical noise plays dual role. It imposes a limitation on sensitivity in the familiar regime of a bifurcation amplifier. On the other hand, it makes it possible to use a bistable modulated oscillator in a new regime of a balanced dynamical bridge. We discuss the switching probabilities and show that they display scaling with control parameters. The critical exponents are found for different types of bifurcations and for different types of noise.

1 Introduction

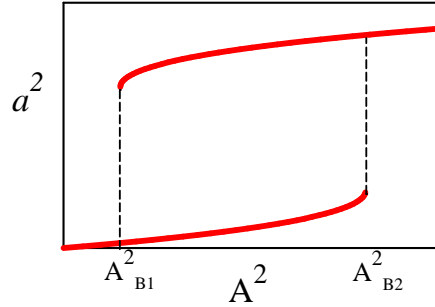
Bistability of vibrational states in modulated systems and fluctuation-induced switching between these states have attracted much attention recently. Experiments have been done on such diverse systems as electrons [1] and atoms [2, 3] in modulated traps, rf-driven Josephson junctions [4, 5], and nano- and micromechanical resonators [6, 7, 8, 9]. These systems have small vibration damping, the quality factor may reach $10^4 - 10^5$. Therefore even a comparatively small resonant field can lead to coexistence of forced vibrations with different phases and amplitudes. This is illustrated in Fig. 1, which refers to a simple model relevant to many of the aforementioned experiments: an underdamped nonlinear classical oscillator driven close to resonance, with equation of motion

Mark Dykman
Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA
e-mail: dykman@pa.msu.edu

$$\ddot{q} + \omega_0^2 q + \gamma q^3 + 2\Gamma \dot{q} = A \cos \omega_F t. \quad (1)$$

Here, q is the oscillator coordinate, ω_0 is its eigenfrequency, Γ is the friction coefficient, $\Gamma \ll \omega_0$, and γ is the nonlinearity parameter. The frequency of the modulating field ω_F is assumed to be close to ω_0 . In this case, for comparatively small modulation amplitude A , even where the oscillator becomes bistable its forced vibrations are nearly sinusoidal, $q(t) = a \cos(\omega_F t + \phi)$ [10].

Fig. 1 Bistability of a nonlinear Duffing oscillator (1). The solid lines show the squared amplitude of forced vibrations as a function of the squared amplitude of the driving force. At the bifurcational values $A_{B1,2}^2$ one of the stable vibrational states disappears.



The dynamic bistability of the oscillator is advantageous for measurements. The idea is to make the oscillator switch between the states depending on the value of the parameter to be measured. Switching leads to a strong change in the system that can be easily detected, leading to a high signal-to-noise ratio in a measurement. This has been successfully used for fast and sensitive measurements of the states of different types of Josephson junction based qubits, including quantum non-demolition measurements [4, 5, 11, 12].

So far the experiments were done in the bifurcation amplifier mode, where the control parameter is swept through a bifurcation point (for example, the field amplitude A was swept through A_{B2} , see Fig. 1). The position of the bifurcation point, i.e., the value of A where switching occurs, depends on the state of the measured qubit. However, instead of happening for a certain A , as expected from Fig. 1, switching happens at random within a certain parameter range of A near A_{B2} .

The randomness of the switching field is a consequence of fluctuations in the oscillator. They lead to switching even before the control parameter reaches its bifurcational value. This is analogous to activated switching out of a potential well studied by Kramers [13]. However, in the case of an oscillator the stable states are not minima of a potential, and there is no static potential barrier that needs to be overcome. Switching of a modulated oscillator is an example of metastable decay of systems far from thermal equilibrium, the phenomenon of a broad interest.

Theoretical analysis of metastable decay requires developing methods for calculating the decay probability and finding out whether decay displays any universal system-independent features, like scaling dependence on the control parameters. For classical systems, scaling of the decay rate was indeed found for systems close to a

bifurcation point, both in the cases of equilibrium [14, 15, 16] and nonequilibrium systems [17, 18, 19]. In the latter case a scaling crossover may occur as the system goes from the underdamped to overdamped regime while approaching the bifurcation point [20]. Such crossover occurs also for quantum tunneling in equilibrium dissipative systems [21].

In this paper we study decay of metastable vibrational states in quantum dissipative systems close to bifurcation points [22]. This is necessary for understanding the operation of a modulated oscillator in the regime of a bifurcation amplifier. We show that at low temperatures decay occurs via quantum activation. This is a specific process that has no analog in systems in thermal equilibrium [23, 24]. As tunneling, quantum activation is due to quantum fluctuations, but as thermal activation, it involves diffusion over an effective barrier separating the metastable state. As we show, near a bifurcation point quantum activation is more probable than tunneling even for $T \rightarrow 0$. We find that the decay rate W scales with the distance to the bifurcation point η as $|\ln W| \propto \eta^\xi$. The scaling exponent is $\xi = 3/2$ for resonant driving, cf. Eq. (1). We also consider parametric resonance in a nonlinear oscillator and show that in this case $\xi = 2$. In addition, $|\ln W|$ displays a characteristic temperature dependence.

2 Quantum Kinetic Equation for a Resonantly Driven Oscillator

The Hamiltonian of a resonantly driven nonlinear oscillator is

$$H_0(t) = \frac{1}{2}p^2 + \frac{1}{2}\omega_0^2 q^2 + \frac{1}{4}\gamma q^4 - qA \cos(\omega_F t). \quad (2)$$

The notations are the same as in equation of motion (1), p is the oscillator momentum. We assume that the detuning $\delta\omega = \omega_F - \omega_0$ of the modulation frequency ω_F from the oscillator eigenfrequency ω_0 is small and that $\gamma \delta\omega > 0$, which is necessary for bistability; for concreteness we set $\gamma > 0$.

It is convenient to switch from q, p to slowly varying operators Q, P , using a transformation

$$q = C_{\text{res}}(Q \cos \omega_F t + P \sin \omega_F t), \quad p = -C_{\text{res}}\omega_F(Q \sin \omega_F t - P \cos \omega_F t) \quad (3)$$

with $C_{\text{res}} = (8\omega_F \delta\omega / 3\gamma)^{1/2}$. The variables Q, P are the scaled coordinate and momentum in the rotating frame. They are canonically conjugate,

$$[P, Q] = -i\lambda, \quad \lambda = 3\hbar\gamma / 8\omega_F^2 \delta\omega. \quad (4)$$

The parameter λ plays the role of the effective Planck constant. We are interested in the semiclassical case; λ is the small parameter of the theory, $\lambda \ll 1$.

In the rotating wave approximation the Hamiltonian (2) becomes $H_0 = (\hbar/\lambda)\delta\omega \hat{g}$, with

$$\hat{g} \equiv g(Q, P) = \frac{1}{4}(Q^2 + P^2 - 1)^2 - \beta^{1/2}Q, \quad \beta = 3\gamma A^2/32\omega_F^3 (\delta\omega)^3 \quad (5)$$

(in the case $\gamma, \delta\omega < 0$ one should use $g \rightarrow -g, H_0 \rightarrow -(\hbar/\lambda)\delta\omega g$). The function g is shown in Fig. 2. It plays the role of the oscillator Hamiltonian in dimensionless time $\tau = t|\delta\omega|$. The eigenvalues of \hat{g} give oscillator quasienergies. The parameter β in Eq. (5) is the scaled intensity of the driving field. For weak damping the oscillator is bistable provided $0 < \beta < 4/27$. The Heisenberg equation of motion for an arbitrary operator M is $\dot{M} \equiv dM/d\tau = -i\lambda^{-1}[M, g]$.

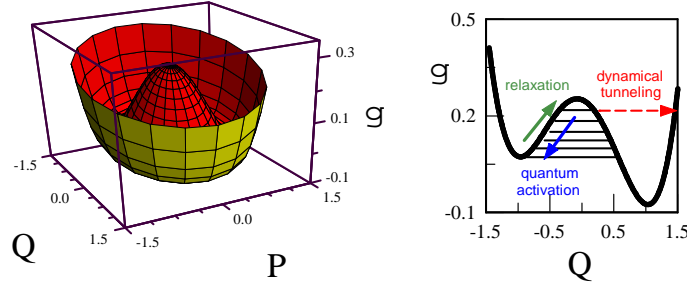


Fig. 2 Quasienergy of a resonantly driven nonlinear oscillator $g(Q, P)$ (left panel) and its cross-section by the plane $P = 0$ (right panel). The plot refers to $\beta = 1/270$ in Eq. (5). Thin horizontal lines in the right panel show (schematically) quasienergy levels for quantized motion around the local maximum of $g(Q, P)$. In the presence of dissipation the states at the local maximum and the minimum of $g(Q, P)$ become stable. They correspond, respectively, to forced vibrations with small and large amplitude a in Fig. 1. The arrows in the right panel show relaxation to the state of small-amplitude vibrations, tunneling from this state with constant quasienergy g , and quantum activation. The latter corresponds to quantum diffusion over quasienergy away from the metastable state, which accompanies relaxation [23, 24].

We will consider two major relaxation mechanisms of the oscillator: damping due to coupling to a thermal bath and dephasing due to oscillator frequency modulation by an external noise. Usually the most important damping mechanism is transitions between neighboring oscillator energy levels. They result from coupling linear in the oscillator coordinate. Since the energy transfer is $\approx \hbar\omega_0$, in the rotating frame the transitions look instantaneous. Phenomenologically, the resulting relaxation may be described by a friction force proportional to velocity, as in (1). Microscopically, such description applies in the case of Ohmic dissipation, i.e., coupling to Ohmic bath. However, we do not have to assume that dissipation is Ohmic. The only assumption needed for the further analysis is that the density of states of the reservoir weighted with the interaction be smooth in the frequency range, which is centered at ω_0 and has a width that largely exceeds $\Gamma, |\delta\omega|$.

We will assume that the correlation time of the noise that modulates the oscillator frequency is also short compared to $1/|\delta\omega|$, so that the noise is effectively

δ -correlated in slow time τ . Then the quantum kinetic equation is Markovian in the rotating frame. It has a familiar form (cf. [25])

$$\dot{\rho} \equiv \partial_\tau \rho = i\lambda^{-1}[\rho, \hat{g}] - \hat{\Gamma}\rho - \hat{\Gamma}^{\text{ph}}\rho, \quad (6)$$

where $\hat{\Gamma}\rho$ describes damping,

$$\hat{\Gamma}\rho = \Gamma|\delta\omega|^{-1} [(\bar{n}+1)(\hat{a}^\dagger\hat{a}\rho - 2\hat{a}\rho\hat{a}^\dagger + \rho\hat{a}^\dagger\hat{a}) + \bar{n}(\hat{a}\hat{a}^\dagger\rho - 2\hat{a}^\dagger\rho\hat{a} + \rho\hat{a}\hat{a}^\dagger)], \quad (7)$$

and $\hat{\Gamma}^{\text{ph}}\rho$ describes dephasing,

$$\hat{\Gamma}^{\text{ph}}\rho = \Gamma^{\text{ph}}|\delta\omega|^{-1} [\hat{a}^\dagger\hat{a}, [\hat{a}^\dagger\hat{a}, \rho]]. \quad (8)$$

Here, Γ and Γ^{ph} are the damping and dephasing rates, $\hat{a} = (2\lambda)^{-1/2}(Q + iP)$ is the oscillator lowering operator, and $\bar{n} = [\exp(\hbar\omega_0/kT) - 1]^{-1}$ is the oscillator Planck number. In the classical mean-field limit one can obtain from (6), (7) the same equation of motion as Eq. (1) in the rotating wave approximation.

In what follows we use dimensionless parameters

$$\Omega = |\delta\omega|/\Gamma, \quad \kappa^{\text{ph}} = \Gamma^{\text{ph}}/\lambda\Gamma. \quad (9)$$

We assume that $\kappa^{\text{ph}} \lesssim 1$. This means that the intensity of phase fluctuations may be comparable to the intensity of quantum fluctuations associated with damping, which is $\propto \lambda\Gamma$, see below, but that $\Gamma^{\text{ph}} \ll \Gamma$.

Metastable decay was studied earlier for additively and parametrically driven oscillators at $T = \kappa^{\text{ph}} = 0$ where there is detailed balance [26, 27, 28], and the lowest nonzero eigenvalue of ρ was studied numerically in Ref. [29]. However, the $T = \kappa^{\text{ph}} = 0$ solution is fragile. It can change exponentially strongly already for extremely small T, κ^{ph} [23, 24]. The analysis [23, 24] revealed the mechanism of quantum activation over a quasienergy barrier, but the results referred to the case where the damping-induced broadening of quasienergy levels is small compared to the typical interlevel distance. This condition necessarily breaks sufficiently close to a bifurcation point where the level spacing becomes small as a consequence of the motion slowing down.

3 Wigner Representation

The analysis of metastable decay near a bifurcation point can be conveniently done in the Wigner representation,

$$\rho_W(Q, P) = \int d\xi e^{-i\xi P/\lambda} \rho \left(Q + \frac{1}{2}\xi, Q - \frac{1}{2}\xi \right), \quad (10)$$

where $\rho(Q_1, Q_2) = \langle Q_1 | \rho | Q_2 \rangle$ is the density matrix in the coordinate representation. Using Eqs. (4)-(10) one can formally write the equation for ρ_W as a sum of terms proportional to different powers of the effective Planck constant λ ,

$$\dot{\rho}_W = -\nabla \cdot (\mathbf{K}\rho_W) + \lambda \hat{L}^{(1)}\rho_W + \lambda^2 \hat{L}^{(2)}\rho_W, \quad (11)$$

where $\mathbf{K} = (K_Q, K_P)$ and $\nabla = (\partial_Q, \partial_P)$. Vector \mathbf{K} determines evolution of the density matrix in the absence of quantum and classical fluctuations,

$$K_Q = \partial_P g - \Omega^{-1}Q \quad K_P = -\partial_Q g - \Omega^{-1}P. \quad (12)$$

This evolution corresponds to classical motion

$$\dot{Q} = K_Q, \quad \dot{P} = K_P. \quad (13)$$

The condition $\mathbf{K} = \mathbf{0}$ gives the values of Q, P at the stationary states of the oscillator in the rotating frame.

The term $\hat{L}^{(1)}$ in (11) describes classical and quantum fluctuations due to damping and dephasing,

$$\hat{L}^{(1)} = \Omega^{-1} \left[\left(\bar{n} + \frac{1}{2} \right) \nabla^2 + \kappa^{\text{ph}} (Q\partial_P - P\partial_Q)^2 \right]. \quad (14)$$

These fluctuations lead to diffusion in (Q, P) -space, as seen from the structure of $\hat{L}^{(1)}$: this operator is quadratic in ∂_Q, ∂_P .

The term $\hat{L}^{(2)}$ in (11) describes quantum effects of motion of the isolated oscillator,

$$\hat{L}^{(2)} = -\frac{1}{4} (Q\partial_P - P\partial_Q) \nabla^2. \quad (15)$$

In contrast to $\hat{L}^{(1)}$, the operator $\hat{L}^{(2)}$ contains third derivatives. Generally the term $\lambda^2 \hat{L}^{(2)}\rho_W$ is not small, because ρ_W varies on distances $\sim \lambda$. However, it becomes small close to bifurcation points, as shown below.

3.1 Vicinity of a Bifurcation Point

From (12), (13), for given reduced damping Ω^{-1} the oscillator has two stable and one unstable stationary state in the rotating frame (periodic states of forced vibrations) in the range $\beta_B^{(1)}(\Omega) < \beta < \beta_B^{(2)}(\Omega)$ and one stable state outside this range [10], with

$$\beta_B^{(1,2)} = \frac{2}{27} \left[1 + 9\Omega^{-2} \mp (1 - 3\Omega^{-2})^{3/2} \right]. \quad (16)$$

At $\beta_B^{(1)}$ and $\beta_B^{(2)}$ the stable states with large and small $Q^2 + P^2$, respectively (large and small vibration amplitudes), merge with the saddle state (saddle-node bifurca-

tion). The values of Q, P at the bifurcation points 1, 2 are

$$Q_B = \beta_B^{-1/2} Y_B (Y_B - 1), \quad P_B = \beta_B^{-1/2} \Omega^{-1} Y_B \quad (Y_B = Q_B^2 + P_B^2), \quad (17)$$

with

$$Y_B^{(1,2)} = \frac{1}{3} \left[2 \pm (1 - 3\Omega^{-2})^{1/2} \right]. \quad (18)$$

In the absence of fluctuations the dynamics of a classical system near a saddle-node bifurcation point is controlled by one slow variable [30]. In our case it can be found by expanding $K_{Q,P}$ in $\delta Q = Q - Q_B$, $\delta P = P - P_B$, and the distance to the bifurcation point $\eta = \beta - \beta_B$. The function K_P does not contain linear terms in $\delta Q, \delta P$. Then, from (13), P slowly varies in time for small $\delta Q, \delta P, \eta$.

On the other hand

$$K_Q \approx -2\Omega^{-1} (\delta Q - a_B \delta P), \quad a_B = \Omega(2Y_B - 1). \quad (19)$$

Therefore the relaxation time of Q is $\Omega/2$, it does not depend on the distance to the bifurcation point. As a consequence, Q follows P adiabatically, i.e., over time $\sim \Omega$ it adjusts to the instantaneous value of P .

4 Metastable Decay near a Bifurcation Point

The adiabatic approximation can be applied also to fluctuating systems, and as we show it allows finding the rate of metastable decay. The approach is well known for classical systems described by the Fokker-Planck equation [31]. It can be extended to the quantum problem by factoring ρ_W into a normalized Gaussian distribution over $\delta\tilde{Q} = \delta Q - a_B \delta P$ and a function $\bar{\rho}_W(\delta P)$ that describes the distribution over δP ,

$$\rho_W \approx \text{const} \times \exp \left[-2 \frac{(\delta Q - a_B \delta P)^2}{\lambda (2\bar{n} + 1) (1 + a_B^2)} \right] \bar{\rho}_W(\delta P).$$

In the spirit of the adiabatic approximation, $\bar{\rho}_W$ can be calculated disregarding small fluctuations of $\delta\tilde{Q}$. Formally, one obtains an equation for $\bar{\rho}_W$ by substituting the factorized distribution into the full kinetic equation (11) and integrating over $\delta\tilde{Q}$. This gives

$$\dot{\bar{\rho}}_W \approx \partial_P [\bar{\rho}_W \partial_P U + \lambda \mathcal{D}_B \partial_P \bar{\rho}_W], \quad (20)$$

where U and \mathcal{D} have the form

$$U = \frac{1}{3} b (\delta P)^3 - \frac{1}{2} \beta_B^{-1/2} \eta \delta P, \quad \eta = \beta - \beta_B,$$

$$\mathcal{D}_B = \Omega^{-1} \left[\left(\bar{n} + \frac{1}{2} \right) + \frac{1}{2} \kappa^{\text{ph}} (1 - Y_B) \right] \quad (21)$$

with

$$b = \frac{1}{2} \beta_B^{1/2} \Omega^2 (3Y_B - 2).$$

In (20), (21) we kept only the lowest order terms in $\delta P, \beta - \beta_B, \lambda$. In particular we dropped the term $-\lambda^2 Q_B \partial_{\bar{p}}^3 \bar{\rho}_W / 4$ which comes from the operator $\hat{L}^{(2)}$ in Eq. (11). One can show that, for typical $|\delta P| \sim |\eta|^{1/2}$, this term leads to corrections $\sim \eta, \lambda$ to $\bar{\rho}_W$.

Eq. (20) has a standard form of the equation for classical diffusion in a potential $U(\delta P)$, with diffusion coefficient $\lambda \mathcal{D}_B$. However, in the present case the diffusion is due to quantum processes and the diffusion coefficient is $\propto \hbar$ for $T \rightarrow 0$.

4.1 Scaling of the Rate of Metastable Decay

For $\eta b > 0$ the potential U (21) has a minimum and a maximum. They correspond to the stable and saddle states of the oscillator, respectively. The distribution ρ_W has a diffusion-broadened peak at the stable state. Diffusion also leads to escape from the stable state, i.e., to metastable decay. The decay rate W is given by the Kramers theory [13],

$$W = C e^{-R_A/\lambda}, \quad R_A = \frac{2^{1/2} |\eta|^{3/2}}{3 \mathcal{D}_B |b|^{1/2} \beta_B^{3/4}}, \quad (22)$$

with prefactor $C = \pi^{-1} (b\eta/2)^{1/2} \beta_B^{-1/4} |\delta\omega|$ (in unscaled time t).

The rate (22) displays activation dependence on the effective Planck constant λ . The characteristic quantum activation energy R_A scales with the distance to the bifurcation point $\eta = \beta - \beta_B$ as $\eta^{3/2}$. This scaling is independent of temperature. However, the factor \mathcal{D}_B in R_A displays a characteristic T dependence. In the absence of dephasing we have $\mathcal{D}_B = 1/2\Omega$ for $\bar{n} \ll 1$, whereas $\mathcal{D}_B = kT/\hbar\omega_0\Omega$ for $\bar{n} \gg 1$. In the latter case the expression for W coincides with the result [17].

In the limit $\Omega \gg 1$ the activation energy (22) for the small-amplitude state has the same form as in the range of β still close but further away from the bifurcation point, where the distance between quasienergy levels largely exceeds their width [23]. We note that the rate of tunneling decay for this state is exponentially smaller. The tunneling is shown by the dashed line in the right panel of Fig. 2. The tunneling exponent for constant quasienergy scales as $\eta^{5/4}$ [18, 32], which is parametrically larger than $\eta^{3/2}$ for small η [for comparison, for a particle in a cubic potential (21) the tunneling exponent in the strong-damping limit scales as η [21]].

For the large-amplitude state the quantum activation energy (22) displays different scaling from that further away from the bifurcation point, where $R_A \propto \beta^{1/2}$ for

$\Omega \gg 1$ [23]. For this state we therefore expect a scaling crossover to occur with varying β .

5 Parametrically Modulated Oscillator

The approach to decay of vibrational states can be extended to a parametrically modulated oscillator that displays parametric resonance. The Hamiltonian of such an oscillator is

$$H_0(t) = \frac{1}{2}p^2 + \frac{1}{2}q^2 [\omega_0^2 + F \cos(\omega_F t)] + \frac{1}{4}\gamma q^4. \quad (23)$$

When the modulation frequency ω_F is close to $2\omega_0$ the oscillator may have two stable states of vibrations at frequency $\omega_F/2$ (period-two states) shifted in phase by π [10]. For $F \ll \omega_0^2$ the oscillator dynamics is characterized by the dimensionless frequency detuning μ , effective Planck constant λ , and relaxation time ζ ,

$$\mu = \frac{\omega_F(\omega_F - 2\omega_0)}{F}, \quad \lambda = \frac{3|\gamma|\hbar}{F\omega_F}, \quad \zeta = \frac{F}{2\omega_F\Gamma}. \quad (24)$$

As before, λ will be the small parameter of the theory.

Parametric excitation requires that the modulation be sufficiently strong, $\zeta > 1$. For such ζ the bifurcation values of μ are

$$\mu_B^{(1,2)} = \mp(1 - \zeta^{-2})^{1/2}, \quad \zeta > 1. \quad (25)$$

If $\gamma > 0$, as we assume, for $\mu < \mu_B^{(1)}$ the oscillator has one stable state; the amplitude of vibrations at $\omega_F/2$ is zero. As μ increases and reaches $\mu_B^{(1)}$ this state becomes unstable and there emerge two stable period two states, which are close in phase space for small $\mu - \mu_B^{(1)}$ (a supercritical pitchfork bifurcation). These states remain stable for larger μ . In addition, when μ reaches $\mu_B^{(2)}$ the zero-amplitude state also becomes stable (a subcritical pitchfork bifurcation). The case $\gamma < 0$ is described by replacing $\mu \rightarrow -\mu$.

The classical fluctuation-free dynamics for μ close to μ_B is controlled by one slow variable [30]. The analysis analogous to that for the resonant case shows that, in the Wigner representation, fluctuations are described by one-dimensional diffusion in a potential, which in the present case is quartic in the slow variable. The probability W of switching between the period-two states for small $\mu - \mu_B^{(1)}$ and the decay probability of the zero-amplitude state for small $\mu - \mu_B^{(2)}$ have the form

$$W = C \exp(-R_A/\lambda), \quad R_A = |\mu_B| \eta^2 / 2(2\bar{n} + 1), \quad \eta = \mu - \mu_B \quad (26)$$

($\mu_B = \mu_B^{(1,2)}$). The corresponding prefactors are $C_B^{(2)} = 2C_B^{(1)} = 2^{1/2}\pi^{-1}\Gamma\zeta^2|\mu_B||\mu - \mu_B|$. Interestingly, dephasing does not affect the decay rate, to the lowest order in $\mu - \mu_B$. This is a remarkable feature of quantum activation near bifurcation points at parametric resonance.

From (26), at parametric resonance the quantum activation energy R_A scales with the distance to the bifurcation point as η^2 . In the limit $\zeta \gg 1$ the same expression as (26) describes switching between period-two states still close but further away from the bifurcation point, where the distance between quasienergy levels largely exceeds their width. In contrast, the exponent for tunneling decay in this case scales as $\eta^{3/2}$ [24].

6 Balanced Dynamical Bridge for Quantum Measurements

Dynamic bistability of a resonantly driven oscillator can be used for quantum measurements in yet another way, which is based on a balanced bridge approach. As a consequence of interstate transitions there is ultimately formed a stationary distribution over coexisting stable states of forced vibrations. The ratio of the state populations w_1, w_2 is

$$w_1/w_2 = W_{21}/W_{12} \propto \exp[-(R_{A2} - R_{A1})/\lambda], \quad (27)$$

where W_{mn} is the switching rate $m \rightarrow n$. For most parameter values $|R_{A1} - R_{A2}| \gg \lambda$, and then only one state is predominantly occupied. However, for a certain relation between β and Ω , where $R_{A1} \approx R_{A2}$, the populations of the two states become equal to each other. This is a kinetic phase transition [33]. A number of unusual effects related to this transition have been observed in recent experiments for the case where fluctuations were dominated by classical noise [9, 34, 35].

In the regime of the kinetic phase transition the oscillator acts as a balanced dynamical bridge: the populations are almost equal with no perturbation, but any perturbation that imbalances the activation energies leads to a dramatic change of $w_{1,2}$, making one of them practically equal to zero and the other to 1. Such a change can be easily detected using, for example, the same detection technique as in the bifurcation amplifier regime. An interesting application of a bridge is measurement of the statistics of white noise in quantum devices. The Gaussian part of the noise does not move a classical oscillator away from the kinetic phase transition. In contrast, the non-Gaussian part, which is extremely important and extremely hard to measure, does, and therefore can be quantitatively characterized.

7 Discussion of Results

It follows from the above results that, both for resonant and parametric modulation, close to bifurcation points decay of metastable vibrational states occurs via quantum activation. The quantum activation energy R_A scales with the distance to the bifurcation point η as $R_A \propto \eta^\xi$, with $\xi = 3/2$ for resonant driving and $\xi = 2$ for parametric resonance. The activation energy R_A is smaller than the tunneling exponent. Near bifurcation points these quantities become parametrically different and scale as different powers of η , with the scaling exponent for tunneling ($5/4$ and $3/2$ for resonant driving and parametric resonance, respectively) being always smaller than for quantum activation.

The exponent of the decay rate R_A/λ displays a characteristic dependence on temperature. In the absence of dephasing, for $kT \gg \hbar\omega_0$ we have standard thermal activation, $R_A \propto 1/T$. The low-temperature limit is described by the same expression with kT replaced by $\hbar\omega_0/2$.

Our results show that quantum activation is a characteristic quantum feature of metastable decay of vibrational states. Activated decay may not be eliminated by lowering temperature. It imposes a limit on the sensitivity of bifurcation amplifiers based on modulated Josephson oscillators used for quantum measurements [11, 12]. At the same time, an advantageous feature of dynamically bistable detectors is that they can be conveniently controlled by changing the amplitude and frequency of the modulating signal. Therefore the value of the activation energy may be increased by adjusting these parameters. Another advantageous feature of these detectors, which has made them so attractive, is that they operate at high frequency and have a small response time, which can also be controlled. The results of this work apply to currently studied Josephson junctions, where quantum regime is within reach. They apply also to nano- and micromechanical resonators that can be used for high-sensitivity measurements in the regime of a balanced dynamical bridge.

Acknowledgements I am grateful to M. Devoret for the discussion. This research was supported in part by the NSF through grant No. PHY-0555346 and by the ARO through grant No. W911NF-06-1-0324.

References

1. L.J. Lapidus, D. Enzer, G. Gabrielse, Phys. Rev. Lett. **83**(5), 899 (1999)
2. R. Gommers, P. Douglas, S. Bergamini, M. Goonasekera, P.H. Jones, F. Renzoni, Phys. Rev. Lett. **94**(14), 143001 (2005)
3. K. Kim, M.S. Heo, K.H. Lee, K. Jang, H.R. Noh, D. Kim, W. Jhe, Phys. Rev. Lett. **96**(15), 150601 (2006)
4. I. Siddiqi, R. Vijay, F. Pierre, C.M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, M.H. Devoret, Phys. Rev. Lett. **93**(20), 207002 (2004)
5. A. Lupaşcu, E.F.C. Driessen, L. Roschier, C.J.P.M. Harmans, J.E. Mooij, Phys. Rev. Lett. **96**(12), 127003 (2006)
6. J.S. Aldridge, A.N. Cleland, Phys. Rev. Lett. **94**(15), 156403 (2005)

7. R.L. Badzey, G. Zolfagharkhani, A. Gaidarzhy, P. Mohanty, *Appl. Phys. Lett.* **86**(2), 023106 (2005)
8. C. Stambaugh, H.B. Chan, *Phys. Rev. B* **73**, 172302 (2006)
9. R. Almog, S. Zaitsev, O. Shtempluck, E. Buks, *Appl. Phys. Lett.* **90**(1), 013508 (2007)
10. L.D. Landau, E.M. Lifshitz, *Mechanics*, 3rd edn. (Elsevier, Amsterdam, 2004)
11. I. Siddiqi, R. Vijay, M. Metcalfe, E. Boaknin, L. Frunzio, R.J. Schoelkopf, M.H. Devoret, *Phys. Rev. B* **73**(5), 054510 (2006)
12. A. Lupaşcu, S. Saito, T. Picot, P.C. De Groot, C.J.P.M. Harmans, J.E. Mooij, *Nature Physics* **3**(2), 119 (2007)
13. H. Kramers, *Physica (Utrecht)* **7**, 284 (1940)
14. J. Kurkijärvi, *Phys. Rev. B* **6**, 832 (1972)
15. R. Victora, *Phys. Rev. Lett.* **63**, 457 (1989)
16. A. Garg, *Phys. Rev. B* **51**(21), 15592 (1995)
17. M.I. Dykman, M.A. Krivoglaz, *Physica A* **104**(3), 480 (1980)
18. A.P. Dmitriev, M.I. Dyakonov, *Zh. Eksp. Teor. Fiz.* **90**(4), 1430 (1986)
19. O.A. Tretiakov, K.A. Matveev, *Phys. Rev. B* **71**(16), 165326 (2005)
20. M.I. Dykman, I.B. Schwartz, M. Shapiro, *Phys. Rev. E* **72**(2), 021102 (2005)
21. A.O. Caldeira, A.J. Leggett, *Ann. Phys. (N.Y.)* **149**(2), 374 (1983)
22. M.I. Dykman, *Phys. Rev. E* **75**(1), 011101 (2007)
23. M.I. Dykman, V.N. Smelyansky, *Zh. Eksp. Teor. Fiz.* **94**(9), 61 (1988)
24. M. Marthaler, M.I. Dykman, *Phys. Rev. A* **73**(4), 042108 (2006)
25. M.I. Dykman, M.A. Krivoglaz, *Soviet Physics Reviews* (Harwood Academic, New York, 1984), vol. 5, pp. 265–441
26. P.D. Drummond, D.F. Walls, *J. Phys. A* **13**(2), 725 (1980)
27. P.D. Drummond, P. Kinsler, *Phys. Rev. A* **40**(8), 4813 (1989)
28. G.Y. Kryuchkyan, K.V. Kheruntsyan, *Opt. Commun.* **127**(4-6), 230 (1996)
29. K. Vogel, H. Risken, *Phys. Rev. A* **38**(5), 2409 (1988)
30. J. Guckenheimer, P. Holmes, *Nonlinear Oscillators, Dynamical Systems and Bifurcations of Vector Fields* (Springer-Verlag, New York, 1987)
31. H. Haken, *Synergetics: Introduction and Advanced Topics* (Springer-Verlag, Berlin, 2004)
32. I. Serban, F. Wilhelm, *Phys. Rev. Lett.* **99**(13), 137001 (2007)
33. M.I. Dykman, M.A. Krivoglaz, *Zh. Eksp. Teor. Fiz.* **77**(1), 60 (1979)
34. C. Stambaugh, H.B. Chan, *Phys. Rev. Lett.* **97**(11), 110602 (2006)
35. H.B. Chan, C. Stambaugh, *Phys. Rev. B* **73**, 224301 (2006)